B Solutions

1. (a) We have

$$\int \cos^4 x\,dx = \int \left(\cos^2 x\right)^2 dx = \int \left(\frac{1}{2}\right)^2 [1+\cos(2x)]^2\,dx = \frac{1}{4}\int \left[1 + 2\cos(2x) + \cos^2(2x)\right] dx$$
$$= \frac{1}{4}\int 1\,dx + \frac{1}{2}\int \cos(2x)\,dx + \frac{1}{4}\int \cos^2(2x)\,dx$$
$$= \frac{1}{4}x + \frac{1}{4}\sin(2x) + \frac{1}{8}\int [1+\cos(4x)]\,dx = \frac{1}{4}x + \frac{1}{4}\sin(2x) + \frac{1}{8}\left[x + \frac{1}{4}\sin(4x)\right] + C$$
$$= \frac{3}{8}x + \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x) + C.$$

(b) This is one for which we use integration by parts:

$$\int (3x-1)e^{-2x}\,dx = -\frac{1}{2}(3x-1)e^{-2x} + \frac{3}{2}\int e^{-2x}\,dx \qquad (\text{with } u = 3x-1,\ dv = e^{-2x}\,dx)$$
$$= -\frac{1}{2}(3x-1)e^{-2x} - \frac{3}{4}e^{-2x} + C.$$

(c) Here,

$$\int \tan^3(3x)\sec^5(3x)\,dx = \int \tan^2(3x)\sec^4(3x)\ \sec(3x)\tan(3x)\,dx$$
$$= \int \left[\sec^2(3x) - 1\right]\sec^4(3x)\ \sec(3x)\tan(3x)\,dx \qquad (\text{using } 1+\tan^2\theta = \sec^2\theta)$$
$$= \frac{1}{3}\int (u^2-1)u^4\,du \qquad (\text{substituting } u = \sec(3x))$$
$$= \frac{1}{21}u^7 - \frac{1}{15}u^5 + C = \frac{1}{21}\sec^7(3x) - \frac{1}{15}\sec^5(3x) + C.$$

(d) Using the substitution $u = x+2$, we get

$$\int_2^7 \frac{x}{\sqrt{x+2}}\,dx = \int_4^9 (u-2)u^{-1/2}\,du = \int_4^9 \left(u^{1/2} - 2u^{-1/2}\right) du = \frac{2}{3}u^{3/2} - 4u^{1/2}\Big|_4^9$$
$$= \frac{2}{3}(27) - 4(3) - \left[\frac{2}{3}(8) - 4(2)\right] = 18 - 12 - \frac{16}{3} + 8 = \frac{42}{3} - \frac{16}{3} = \frac{26}{3}.$$

2. The region is horizontally simple, but not vertically simple. We solve the equations for x in preparation for horizontal slices (i.e, slices at fixed y-values with $0 \le y \le 1$):

left boundary: $x = 1 - y$
right boundary: $x = 1 + \sqrt{y}$

Then

$$\text{Area} = \int_0^1 [(1+y^{1/2}) - (1-y)]\,dy = \int_0^1 (y^{1/2} + y)\,dy = \frac{2}{3}y^{3/2} + \frac{1}{2}y^2\Big|_0^1 = \frac{2}{3} + \frac{1}{2} = \frac{7}{6}.$$

3. (a) Since R is horizontally simple, that provides some motivation for making horizontal slices through R which generate cylindrical shells. The lateral "height" of a cylinder at fixed y is the difference, $(y^{1/2} + y)$ after simplifying, of right and left boundary (see the solution to the previous problem). We obtain

$$V = 2\pi \int_0^1 y(y^{1/2} + y)\, dy.$$

Perhaps more difficult, but equally valid, is to slice through R vertically and employ the washer method, yielding the sum of x-integrals

$$V = \int_0^1 \pi[1^2 - (1-x)^2]\, dx + \int_1^2 \pi[1^2 - (x-1)^4]\, dx = \pi \int_0^1 (2x - x^2)\, dx + \pi \int_1^2 (4x - 6x^2 + 4x^3 - x^4)\, dx.$$

(b) Using the method of washers (this time a y-integral), we have

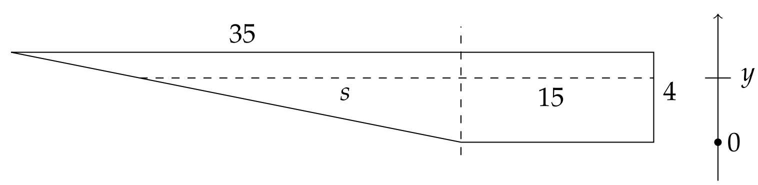
$$V = \pi \int_0^1 \left(\left[2 - (1-y)\right]^2 - \left[2 - (1 + \sqrt{y})\right]^2 \right) dy = \cdots = \pi \int_0^1 (y^2 + y + 2\sqrt{y})\, dy.$$

If we do this by shells, we get another sum of integrals:

$$\begin{aligned} V &= \int_0^1 2\pi(2-x)[1 - (1-x)]\, dx + \int_1^2 2\pi(2-x)[1 - (x-1)^2]\, dx \\ &= 2\pi \int_0^1 (2x - x^2)\, dx + 2\pi \int_1^2 (2-x)(2x - x^2)\, dx. \end{aligned}$$

4. Viewing the figure, a slice at height y will be a "slab" with length $s + 15$, width 20 and thickness dy. By similar triangles,

$$\frac{s}{35} = \frac{y}{4}, \qquad \text{or} \qquad s = \frac{35}{4} y.$$



Thus, the slab has

$$\begin{aligned} \text{Volume} &= 20\left(\frac{35}{4} y + 15\right) dy \\ \text{Mass} &= 20(1000)\left(\frac{35}{4} y + 15\right) dy \\ \text{Weight} &= 20(9.8)(1000)\left(\frac{35}{4} y + 15\right) dy. \end{aligned}$$

This weight must be lifted from height y to height 4, a distance of $(4 - y)$. So, the desired integral is

$$\text{Work} = \int_0^4 20(9.8)(1000)\left(\frac{35}{4} y + 15\right)(4 - y)\, dy.$$