A Solutions

1. (a) The series $\sum_n |\cos n|/n^{1.1}$ has nonnegative terms which are always smaller than the terms of $\sum_n 1/n^{1.1}$. Since the latter is a convergent p-series, the former converges by the direct comparison test. Thus, our original series converges by the absolute convergence test.

   (b) The series is geometric with $r = \pi/3 > 1$, and hence divergent.

2. (a) Since

$$\frac{x-11}{x^2+3x-4} = \frac{x-11}{(x+4)(x-1)} = \frac{A}{x+4} + \frac{B}{x-1},$$

   we have

$$x - 11 = A(x-1) + B(x+4).$$

   This equation must hold for all x—in particular, at

$$x = 1: \quad \text{giving} \quad 1 - 11 = A(0) + B(1+4) \quad \Rightarrow \quad B = -2,$$

   and at

$$x = -4: \quad \text{giving} \quad -4 - 11 = A(-4-1) + B(0) \quad \Rightarrow \quad A = 3.$$

   Thus, our integral equals

$$\int \frac{3}{x+4}\,dx - \int \frac{2}{x-1}\,dx = 3\ln|x+4| - 2\ln|x-1| + C.$$

   (b) Making the substitution $x = \sin\theta$ (so $dx = \cos\theta\,d\theta$), the integral becomes

$$\begin{aligned}
\int \frac{(\sin^2\theta + 1)\cos\theta}{\sqrt{1-\sin^2\theta}}\,d\theta &= \int \frac{(\sin^2\theta+1)\cos\theta}{\sqrt{\cos^2\theta}}\,d\theta = \int \frac{(\sin^2\theta+1)\cos\theta}{\cos\theta}\,d\theta \\
&= \int (\sin^2\theta + 1)\,d\theta = \int \left(\frac{1}{2}[1-\cos(2\theta)] + 1\right) d\theta \\
&= \int \left[\frac{3}{2} - \frac{1}{2}\cos(2\theta)\right] d\theta = \frac{3}{2}\theta - \frac{1}{4}\sin(2\theta) + C \\
&= \frac{3}{2}\arcsin(x) - \frac{1}{4}\sin(2\arcsin(x)) + C.
\end{aligned}$$

   Note, using the identity $\sin(2\theta) = 2\cos\theta\sin\theta$, one can show that

$$\frac{3}{2}\theta - \frac{1}{4}\sin(2\theta) + C = \frac{3}{2}\theta - \frac{1}{2}\cos\theta\sin\theta + C = \frac{3}{2}\arcsin(x) - \frac{1}{2}x\sqrt{1-x^2} + C,$$

   but the above answer is just fine.

3. We have

$$\sum_{n=1}^{\infty} \frac{3^{n+1}}{2^{2n}} = \frac{9}{4} + \frac{27}{16} + \frac{81}{64} + \cdots = \frac{9}{4}\left[1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \cdots\right],$$

   a geometric series with $r = 3/4$ (thus convergent) whose sum is

$$\frac{9}{4} \cdot \frac{1}{1-3/4} = 9.$$

4. Since $f'(x) = (3/2)x^{1/2}$, we have arc length given by the integral

$$\int_0^5 \frac{1}{2}\sqrt{4+9x}\,dx = \frac{1}{18}\int_4^{49} u^{1/2}\,du = \frac{1}{27}u^{3/2}\Big|_4^{49} = \frac{1}{27}(7^3 - 2^3) = \frac{335}{27} \doteq 12.41.$$

5. We have derivatives

$$\begin{array}{rclcrcl} f^{(0)}(x) &=& x^{1/2} & \Rightarrow & f^{(0)}(4) &=& 2. \\ f'(x) &=& \frac{1}{2}\,x^{-1/2} & \Rightarrow & f'(4) &=& \frac{1}{2\sqrt{4}} = \frac{1}{4}. \\ f''(x) &=& -\frac{1}{4}\,x^{-3/2} & \Rightarrow & f''(4) &=& -\frac{1}{4}\cdot\frac{1}{(\sqrt{4})^3} = -\frac{1}{32}. \end{array}$$

Thus, the desired Taylor polynomial is

$$T_2(x) \;=\; f(4) + f'(4)(x-4) + \frac{f''(4)}{2}(x-4)^2 \;=\; 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2.$$

6. If we first find an antiderivative using the substitution $u = \ln x$ (so $du = dx/x$)

$$\int \frac{dx}{x(\ln x)^{3/2}} \;=\; \int \frac{du}{u^{3/2}} \;=\; \int u^{-3/2}\,du \;=\; -2\,u^{-1/2} + C \;=\; -\frac{2}{(\ln x)^{1/2}} + C,$$

then we have

$$\int_3^\infty \frac{dx}{x(\ln x)^{3/2}} \;=\; \lim_{A\to\infty}\int_3^A \frac{dx}{x(\ln x)^{3/2}} \;=\; \lim_{A\to\infty}\left[-\frac{2}{(\ln x)^{1/2}}\right]_3^A \;=\; \lim_{A\to\infty}\left[\frac{2}{(\ln 3)^{1/2}} - \frac{2}{(\ln A)^{1/2}}\right] \;=\; \frac{2}{(\ln 2)^{1/2}}.$$

Thus, the integral converges (to the value specified).