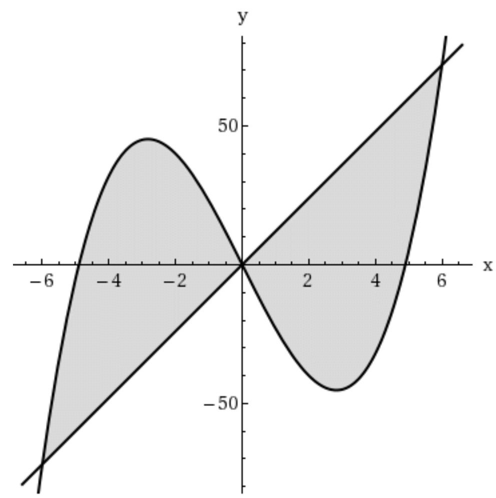
Applications of Integration

1. Find the area of the region enclosed by the graphs of $y = x^3 - 24x$ and $y = 12x$.



2. Sketch a region whose area is represented by the integral $\int_{-2/\sqrt{2}}^{2/\sqrt{2}} \left(\sqrt{4-x^2} - |x|\right) dx$, and evaluate using geometry.

3. Find the volume of the solid whose base is the semicircle $y = \sqrt{25 - x^2}$, where $-5 \leqslant x \leqslant 5$ and whose cross sections perpendicular to the x-axis are squares.

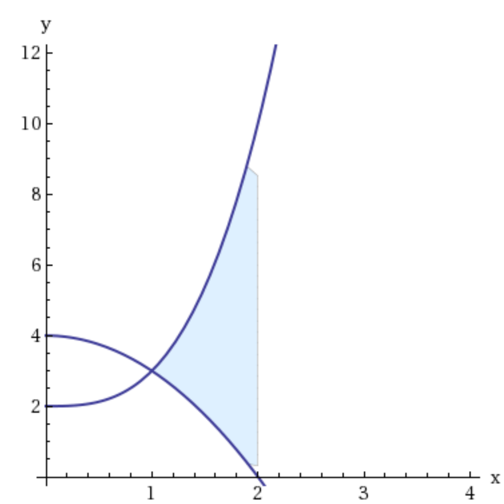
4. An object with zero initial velocity accelerates at a constant rate 4 m/s^2. Find its average velocity during the first 14 s.

5. Find the volume of the solid obtained by rotating the region under the graph of $f(x) = 6x - x^2$ about the x-axis over the interval $[0, 2]$.

6. Find the volume of the solid obtained by rotating the region enclosed by the graphs of $y = 2\sqrt{x}$ and $y = x$ about the line $x = -14$.

7. Find the volume of the solid obtained by rotating the region underneath the graph of $f(x) = 81 - x^2$ about the y-axis over the interval $[1, 9]$.

8. Find the volume of rotation of the region enclosed by the graphs of $y = x^3 + 2$, $y = 4 - x^2$, and $x = 2$, as shown in the figure, about the line $x = 4$.



9. Compute the work required to stretch a spring from 8 cm to 11 cm past equilibrium, assuming that the spring constant is $k = 170$ kg/s^2.

10. A 2-meter chain with linear mass density $\rho(x) = 2x(4-x)$ kg/m, $0 \leqslant x \leqslant 2$, lies on the ground. Calculate the work required to lift the chain from its front end so that its bottom is 2 m above ground. (Assume the chain is lifted from the end at $x = 2$, and take $g = 9.8$ m/s^2.

11. Approximate the arc length of the curve $y = 7e^{-x^2}$ over the interval $[0, 2]$ using Simpson's Rule with 8 subintervals.

12. Find the centroid of the region lying underneath the graph of the function $f(x) = \ln(x^{10})$ over the ihnterval $[1, 2]$.

13. Find the centroid of the region between the x-axis and the top half of the ellipse $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$.