Infinite Series and Power Series

1. Find the Taylor polynomials T_2 and T_3 centered at $x = 1$ for the function $f(x) = \ln x/x$.
2. (a) Use the Maclaurin series of $(1-x)^{-1}$ to write the Maclaurin series for $f(x) = (1+2x^4)^{-1}$.
   (b) Find the Maclaurin polyomials $T_0, T_1, T_2, \ldots T_8$ for the same function f as in the previous part.
3. Determine the limit of the sequence $a_n = 28 - 3/n^{1/2}$, or show that the sequence diverges.
4. Consider the series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{6}{n+2} - \frac{6}{n+3}\right)$.
   (a) Find the partial sums S_3, S_4 and S_5.
   (b) Find the sum of the series, or show that the series diverges.
5. Find the sum of the series $\displaystyle\sum_{n=0}^{\infty}\left(\frac{7\pi}{8e}\right)^n$, or show that the series diverges.
6. Use an appropriate test to determine the fate (convergent or divergent) of the series $\displaystyle\sum_{n=3}^{\infty}\frac{4\ln n}{n^2}$.
7. Determine whether the series $\displaystyle\sum_{n=1}^{\infty}\frac{n^3 - \cos n}{n^5}$ converges or diverges.
8. Determine whether the series $\displaystyle\sum_{n=1}^{\infty}\frac{\sin(n\pi/4)}{n^2}$ *converges absolutely*, *converges conditionally*, or *diverges*.
9. The series $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(n+6)(n+9)}$ *alternates*.
   (a) Explain how we know the series converges.
   (b) Find the smallest (integer) value N for which the partial sum S_N of the series has an error of at most 10^{-5}.
10. Apply the Ratio Test to determine, if possible, the fate of the series $\displaystyle\sum_{n=1}^{\infty}\frac{5\cdot 10^n}{2^{n^2}}$.
11. Apply the Root Test to determine, if possible, the fate of the series $\displaystyle\sum_{k=0}^{\infty}\left(\frac{k}{k+13}\right)^k$.
12. Find the interval of convergence for the power series $\displaystyle\sum_{n=0}^{\infty}\frac{x^n}{n^5+4}$.
13. (a) Find the Maclaurin series for the function $f(x) = \dfrac{9}{1+5x}$.
    (b) What is the interval of convergence for this series?

14. Find the Maclaurin series for $f(x) = x^6 e^{-x^3}$.

15. Find the terms through degree four of the Maclaurin series of $f(x) = [1 + \sin(5x)]^{-1}$.