From ODELA Section 2.5, pp. 125–126, do Exercise 2.5.4.

From ODELA Section 3.2, p. 143, do Exercise 3.2.1, parts (b), (e) and (f).

Consider the 2nd-order system
\[
\frac{d^2 u_1}{dt^2} = \frac{k_2}{m_1} (u_2 - u_1) + \frac{c_2}{m_1} \left( \frac{du_2}{dt} - \frac{du_1}{dt} \right) - \frac{k_1}{m_1} u_1 - \frac{c_1}{m_1} \frac{du_1}{dt},
\]
\[
\frac{d^2 u_2}{dt^2} = \frac{f(t)}{m_2} - \frac{k_2}{m_2} (u_2 - u_1) - \frac{c_2}{m_2} \left( \frac{du_2}{dt} - \frac{du_1}{dt} \right).
\]

which may (it has been proposed by someone on the internet, but I have not verified it) accurately model the mass-spring assembly with dashpots (shock absorbers) pictured at the website

http://upload.wikimedia.org/wikipedia/commons/f/fd/Mass-Spring-Damper_%282_body_system%29.svg

It seems the displacements from equilibrium positions of masses \(m_1, m_2\) are denoted by \(u_1, u_2\) respectively. The spring constants are \(k_1, k_2\), and \(c_1, c_2\) are the damping constants associated with the dashpots. There appears, as well, to be an external force \(f(t)\) applied to the mass on the right.

Convert this model (whether it models what it purports to or not) to a first order linear system of DEs in the form \(x' = Ax + b(t)\), specifying the entries of \(A\) and \(b(t)\).

From ODELA Section 3.6, pp. 181–183, do Exercise 3.6.12.

On pp. 188-189 of ODELA we learn about a 1st order linear DE system model for quantities of lead stored in 1) the blood, 2) body tissues, and 3) bones. The model is
\[
\begin{bmatrix}
x'_1 \\
x'_2 \\
x'_3
\end{bmatrix} = \begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} + \begin{bmatrix}
I_b(t) \\
0 \\
0
\end{bmatrix},
\]
where
\[
A = \begin{bmatrix}
-13/360 & 272/21875 & 7/200000 \\
1/90 & -1/35 & 0 \\
7/1800 & 0 & -7/200000
\end{bmatrix}.
\]

(a) Approximate eigenpairs for the matrix \(A\) are provided at the top of p. 190. Use them to write the homogeneous solution—i.e., the solution in the case the influx into the bloodstream of lead from the environment \(I_b(t) = 0\).

(b) If \(I_b(t) = 0\) for a person previously poisoned with lead, what aspect of the model or solution indicates that the lead will be flushed out over time?
(c) Continue assuming that $I_L(t) = 0$, but suppose we have initial conditions $x_1(0) = 50$, $x_2(0) = 0$ and $x_3(0) = 0$; that is, we start with 50 units of lead in the blood and none in tissue nor bone. Solve the (homogeneous) IVP, and use it to write a formula for the amount $x_3(t)$ of lead in the bones. Find the approximate time $t$ (in days) at which the level of lead in the bones is at its peak value. [Give your answer accurate to the tenths place.] Also, find the approximate time, following that peak, when the lead level in the bones has receded to no more than 0.5 units.