MATH 231: Differential Equations with Linear Algebra

Hand-Checked Assignment #7, due date: none

This is homework that will **not** be collected. Rather, it is to offer you practice problems and, at some point (probably on Sunday), I will post solutions.

★42 From ODELA Section 5.2, pp. 246, do Exercise 5.2.2. Focus particularly on parts (b), (e) and (f).

★43 From ODELA Section 5.2, p. 246, do Exercise 5.2.3.

★44 (a) Consider the IVP

$$y'' + 6y' + 5y = 7e^{2t}, \quad y(0) = \frac{4}{3}, \quad y'(0) = -3.$$

Solve it as you would in Chapter 4. That is, find the homogeneous solution $y_h(t)$, use the method of undetermined coefficients to find $y_p(t)$, then apply the initial conditions.

(b) Use Chapter 4 methods to solve the IVP

$$y'' + 6y' + 5y = 0, \quad y(0) = \frac{4}{3}, \quad y'(0) = -3.$$

If you take this solution (with its choices of c_1, c_2) and add it with the particular solution you found in part (a), does their sum satisfy the IVP of part (a)?

(c) Use Laplace transform methods to solve the IVP

$$y'' + 6y' + 5y = 7e^{2t}, \quad y(0) = 0, \quad y'(0) = 0.$$

The solution you just found can also be called a *particular solution* of $y'' + 6y + 5y = 7e^{2t}$. If you add this solution with the solution to part (b) (with the choices of c_1, c_2 you found in (b)), does the result solve the problem in part (a)?

(d) [Optional: Food for thought.] **Generalization?** In solving IVPs for nonhomogeneous problems, I have witnessed some students solving for the weights c_1, c_2 (c_3, etc.) by applying the initial conditions directly to y_h, before they have tacked on the particular solution. When does this strategy work?

(e) What is the **steady state** solution to part (a)?

★45 From ODELA Section 5.3, p. 250, do Exercises 5.3.1 and 5.3.2. (They are closely related, and for each individual part it should be straightforward to answer both questions at once.)

★46 From ODELA Section 5.5, p. 267, do Exercises 5.5.1 and 5.5.2. (They are closely related, and for each individual part it should be straightforward to answer both questions at once. Note that your answers to 5.5.2 are constrained by the fact that $f(t)$ has not been specified.)