Vectors

n-vector in $\mathbb{R}^n$ with n real-number components

$$\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \in \mathbb{R}^3, \qquad \begin{bmatrix} 2 \\ 7 \\ 1 \\ -3 \end{bmatrix} \in \mathbb{R}^4$$

3×1 $\qquad$ 4×1

$\langle 1, 2, -1 \rangle \in \mathbb{R}^3$

but $\begin{bmatrix} 1 & 2 & -1 \end{bmatrix}$ since it's 1×3 not 3×1.

Vectors of same shape can be added

$$\begin{bmatrix} 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

Can rescale vectors:

$$3 \begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 12 \\ 3 \end{bmatrix}$$

scalar $\quad$ initial

When you
- rescale a single vector, or
- sum of (possibly rescaled) vectors,

its called a <u>linear combination</u>.

Q: Is $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ in <u>the span of</u> the vectors $\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix}$?

translation: Can you take a linear combination of

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \text{ and } \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$ —

a sum of rescaled versions of these vectors

$$c\begin{bmatrix} 1 \\ 2 \end{bmatrix} + d\begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

— so as to get $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$?

We're tasked with finding, if possible, scalars c and d so that

$$\begin{bmatrix} 3 \\ 1 \end{bmatrix} = c\begin{bmatrix} 1 \\ 2 \end{bmatrix} + d\begin{bmatrix} -1 \\ 2 \end{bmatrix}.$$

A: Propose $c = \frac{7}{4}$, $d = \frac{-5}{4}$

So $$\frac{7}{4}\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \frac{-5}{4}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 7/4 \\ 7/2 \end{bmatrix} + \begin{bmatrix} 5/4 \\ -5/2 \end{bmatrix} = \begin{bmatrix} 12/4 \\ 2/2 \end{bmatrix} \checkmark$$