Is $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ in span of $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix} \right\}$?

Looked to see if scalars / weights c, d exist so that

$$c \begin{bmatrix} 1 \\ 2 \end{bmatrix} + d \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Using algebra

$$\begin{bmatrix} 3 \\ 1 \end{bmatrix} = c \begin{bmatrix} 1 \\ 2 \end{bmatrix} + d \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} c \\ 2c \end{bmatrix} + \begin{bmatrix} -d \\ 2d \end{bmatrix}$$

$$= \begin{bmatrix} c - d \\ 2c + 2d \end{bmatrix}$$

Comes down to finding 2 unknowns c, d which satisfy 2 eqns.

$$c - d = 3$$
$$2c + 2d = 1$$

augmented matrix

$$\left. \begin{aligned} c - d &= 3 \\ 2c + 2d &= 1 \end{aligned} \right\} \qquad \left[\begin{array}{cc|c} 1 & -1 & 3 \\ 2 & 2 & 1 \end{array} \right]$$

Subtract $2 \times$ (1st eqn.) from the 2nd

$$\begin{aligned} c - d &= 3 \\ 4d &= -5 \end{aligned} \qquad \left[\begin{array}{cc|c} 1 & -1 & 3 \\ 0 & 4 & -5 \end{array} \right]$$

Divide 2nd eqn. by 4

$$\begin{aligned} c - d &= 3 \\ d &= -5/4 \end{aligned} \qquad \begin{array}{cc} c & d \end{array} \left[\begin{array}{cc|c} 1 & -1 & 3 \\ 0 & 1 & -5/4 \end{array} \right] \rightarrow d = -5/4$$

Add row 2 to row 1

$$\left[\begin{array}{cc|c} 1 & 0 & 7/4 \\ 0 & 1 & -5/4 \end{array}\right] \rightarrow c = 7/4$$

$d = -5/4$

Gaussian Elimination (GE)

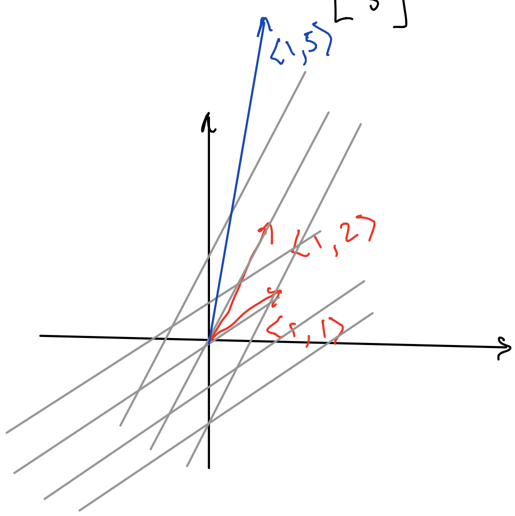
- convert system of equations to augmented matrix
- use "legal moves" (elementary row operations = EROs) to morph our augmented matrix into a desirable form where we can obtain values of unknowns

EROs:

1. Swap two rows: rowswap
2. Multiply a row by a nonzero scalar: mrow, *row
3. Add multiple of one row to another: mrowadd, *row+

Ex.

Q: Is $\begin{bmatrix} 1 \\ 5 \end{bmatrix}$ in span of $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$?



$$x \begin{bmatrix} 1 \\ 1 \end{bmatrix} + y \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

or

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 1 & 2 & 5 \end{array}\right]$$

make look like $\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 1 & 2 & 5 \end{array}\right] \underset{\sim}{(-1)r_1+r_2 \to r_2} \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & 4 \end{array}\right]$$

$$\underset{\sim}{(-1)r_2+r_1 \to r_1} \left[\begin{array}{cc|c} 1 & 0 & -3 \\ 0 & 1 & 4 \end{array}\right]$$

Weights $x = -3$, $y = 4$

Ex.) Write $\begin{bmatrix} -2 \\ 4 \\ 15 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 9 \end{bmatrix}$

if possible, or demonstrate that it cannot be done.

Seek weights

$$x_1 \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 1 \\ 9 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \\ 15 \end{bmatrix}$$

leading to augmented matrix

$$\left[\begin{array}{ccc|c} 3 & -1 & 1 & -2 \\ -1 & 1 & 1 & 4 \\ 5 & 2 & 9 & 15 \end{array}\right]$$