Math 231, Mon 8-Feb-2021 -- Mon 8-Feb-2021
Differential Equations and Linear Algebra
Spring 2020

Monday, February 8th 2021

Wk 2, Mo
Topic:: Matrix-matrix arithmetic
Read:: ODELA 1.3-1.4

Conclusions added to the end of Friday's class notes
Questions/issues over homework
Identifying row echelon forms

echelon form:
- first nonzero entry in each row (pivot) further to right than in previous rows
- all completely-zeroed rows are at bottom

Identify those matrices which are in echelon form.

(a) $\begin{bmatrix} 0 & 0 & 4 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 0 & 4 & 1 \\ 0 & 5 & 0 & 2 \\ -2 & 1 & 0 & 3 \end{bmatrix}$

RREF

(c) $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

$-r_3 + r_2 \to r_2$

(e) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

(f) $\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

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Follow-up
 - What if a 2-by-2 augmented matrix represents a
    consistent system
      What possible echelon form?
      What implication about the two column vectors?
    inconsistent system
      What possible echelon form?
      What implication about the two column vectors?
 - What if a 2-by-3 augmented matrix represents a
    consistent system
    inconsistent system
 - What if a 3-by-3 augmented matrix represents a
    consistent system
    inconsistent system

Matrix algebra
 - adding
 - rescaling
 - multiplication
    define it
    do an example
    implications on dimensions
      may make sense as AB, but not as BA
      ==> commutativity is out
      a necessary (but not sufficient) condition for AB = BA is being square
    identity matrix
 - solving Ax = b by "division"
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    cannot divide matrices
    might conceive of a matrix C for which AC = CA = I
      A would need to be square (C, too)
      If such a C exists, then Ax = b has solution x = Cb
 - Using AC = I and GE to find inverse
    Do for 2-by-2 matrix A = [a b; c d]
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$m \times n$ matrix w/ real entries

adding is component-wise requiring same dimensions

$$\underset{3\times 2}{\begin{bmatrix} 4 & 2 \\ 1 & -1 \\ 2 & 0 \end{bmatrix}} + \underset{3\times 2}{\begin{bmatrix} 1 & 1 \\ -1 & 0 \\ 3 & 1 \end{bmatrix}} = \underset{3\times 2}{\begin{bmatrix} 5 & 3 \\ 0 & -1 \\ 5 & 1 \end{bmatrix}}$$

rescaling

$$2 \underset{3\times 2}{\begin{bmatrix} 4 & 2 \\ 1 & -1 \\ 2 & 0 \end{bmatrix}} = \begin{bmatrix} 8 & 4 \\ 2 & -2 \\ 4 & 0 \end{bmatrix}$$

Multiplying matrices

already have special case of matrix times a vector

$$\underset{3\times 2}{\begin{bmatrix} 4 & 2 \\ 1 & -1 \\ 2 & 0 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} 2 \\ 1 \end{bmatrix}} = 2\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} + 1\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ 1 \\ 4 \end{bmatrix}$$

since a vector

shape matches a single col. in matrix

$\vec{b}_i$'s are columns in B

extend to mult. 2 matrices

$$A\ B = A \begin{bmatrix} \vec{b}_1 & \vec{b}_2 & \cdots & \vec{b}_p \\ \downarrow & \downarrow & & \downarrow \\ & & & \end{bmatrix}$$

$$:= \begin{bmatrix} A\vec{b}_1 & A\vec{b}_2 & \cdots & A\vec{b}_p \\ \downarrow & \downarrow & & \downarrow \\ & & & \end{bmatrix}$$

Ex.)
$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 4 \\ 1 & 0 & -3 \end{bmatrix}_{3\times 3}, \quad B = \begin{bmatrix} 4 & -2 \\ 1 & 2 \\ -2 & 1 \end{bmatrix}_{3\times 2}$$

$$\Rightarrow \vec{b}_1 = \begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix}, \quad \vec{b}_2 = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$$

$$A\vec{b}_1 = 4\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + 1\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} - 2\begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 10 \end{bmatrix}$$

$$A\vec{b}_2 = -2\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + 1\begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -5 \end{bmatrix}$$

$$AB = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}_{3\times 3} \begin{bmatrix} b_1 & b_2 \\ \downarrow & \downarrow \\ & \end{bmatrix}_{3\times 2} = \begin{bmatrix} 8 & 1 \\ 1 & 2 \\ 10 & -5 \end{bmatrix}_{3\times 2}$$

$$\begin{array}{cc} \uparrow & \\ A\vec{b}_1 & A\vec{b}_2 \end{array}$$

$A_{m\times n} \cdot B_{q\times p}$ makes sense only when $n = q$

$A_{3\times \underline{2}} \cdot B_{\underline{2}\times 7}$ results in a 3×7 matrix

but $B_{2\times 7} \cdot A_{3\times 2}$ can't be performed.