Math 231, Wed 3-Mar-2021 -- Wed 3-Mar-2021
Differential Equations and Linear Algebra
Spring 2020

Wednesday, March 3rd 2021

Wk 5, We
Topic:: DE classifications
Topic:: Direction fields

Classifications to note:
- Order
- Autonomous vs. nonautonomous
- linear
    homogeneous vs. nonhomogeneous
    where we'll spend most of our time
- ordinary vs. partial
    comes down to the count of independent vars
- single DE vs. system
    comes down to the count of dependent vars

Direction fields
- possible for 1st-order (single) ODEs which are put into normal form
- usage?

Separable DEs
- resides in the intersection of
    ordinary DEs
    1st-order
    single DE (not a system)
- key feature: can be put in form: M(y) dy/dx = N(x)
    recognizing if a form is possible is key in DEs!
    Note: The DE y' = cos(t) - 2y is NOT separable
- solving
    Can proceed to integrate: \int M(y) dy = \int N(x) dx
    Not always easy to tease out explicit formula for y(x)

Not yet discussed

deriv $3e^{kt} \cdot k$

orig. $3e^{kt}$

basis soln.

Propose soln: $y(t) = C\boxed{e^{kt}}$

scalar

$y' = \frac{dy}{dt}$

Population Models

k constant

A simple model:

1

$$\boxed{y' = ky.} \tag{1}$$

Describe in words like "rate of change of unknown is proportional to the size of the unknown."

Instead of calling this a population, try to elicit from students scenarios like the following that fit our model description:

- population
- investments
- radioactive decay

At some point let on that the general solution is Ce^{kt}, indicating the flaw in such a model for populations.

Pose the alternative model

$$y' = ry(1-y) = ry - ry^2. \tag{2}$$

Note that

- When $y \approx 0$, the nonlinear term ry^2 is negligible in comparison to the linear term ry, so the model should behave like the previous one. Effect of the nonlinear term is felt increasingly as the size of y grows.
- There are **equilibrium solutions** $y \equiv 0$ and $y \equiv 1$.

Direction Fields

To investigate this model further, introduce direction fields.

- Direction fields possible for 1st order DEs which can be arranged in **normal form**

$$y' = f(t, y) \tag{3}$$

- At each point (t, y) sketch a hash mark with slope $f(t, y)$. Practice drawing for the logistic equation above.

  Will expect students to sketch by hand for autonomous DEs.

- Use direction field applet at `http://math.rice.edu/%7edfield/dfpp.html`

links from class webpage

Model:

(2) $$\underline{mv'} = \underbrace{mg}_{\text{gravitational force}} - \overbrace{\gamma v}^{\text{drag force}}$$

falling body

m = mass

v = velocity

$g = 9.8 \ m/s^2$

γ = drag constant

(3) Newton's Law of Cooling

T = body's temperature

T_0 = ambient temperature (const)

k = constant of proportion

$$\frac{dT}{dt} = -k(T - T_0)$$

1st-order ODEs

- Look like $F(y, y', t) = 0$
- Solutions are families of fns., not unique (single) answers

Ex) e^{kt} is <u>not</u> only soln. of $y' = ky$

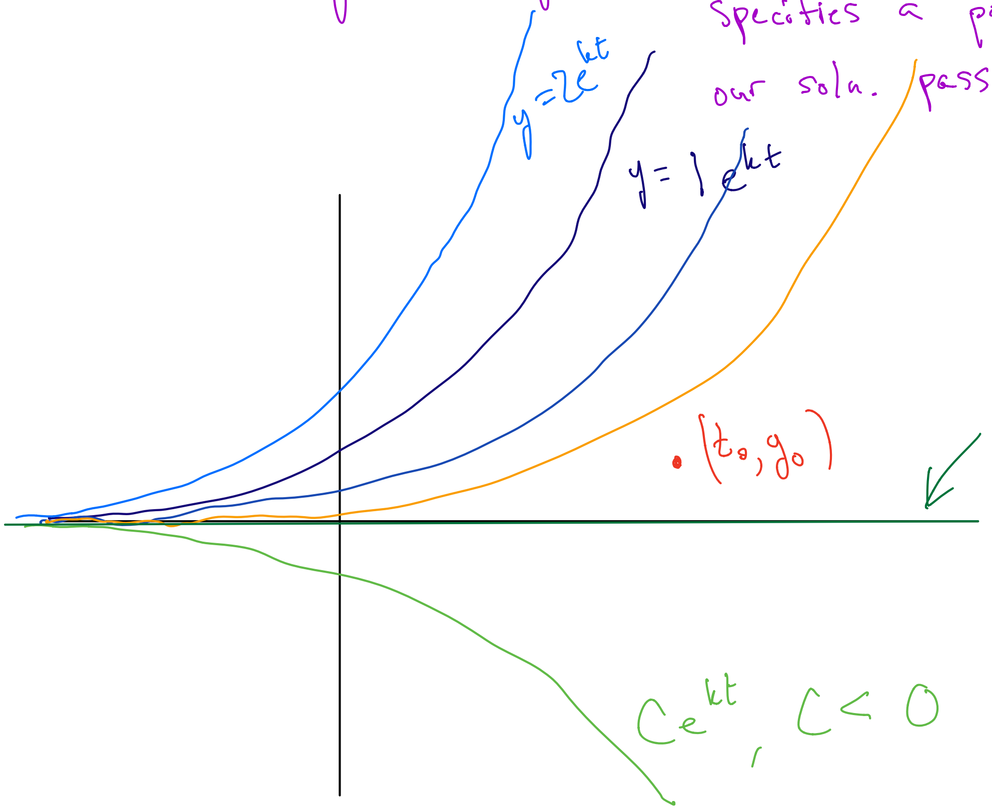
There are others — namely all scalar mults.

To obtain a unique soln., one can solve an IVP (initial value problem)

Ex.

$$y' = ky \quad \text{DE}$$

$$y(t_0) = y_0$$ initial condition specifies a point in ty-plane our soln. passes through



0 soln, when $C = 0$

- Some 1st-order DEs allow for arranging in the form

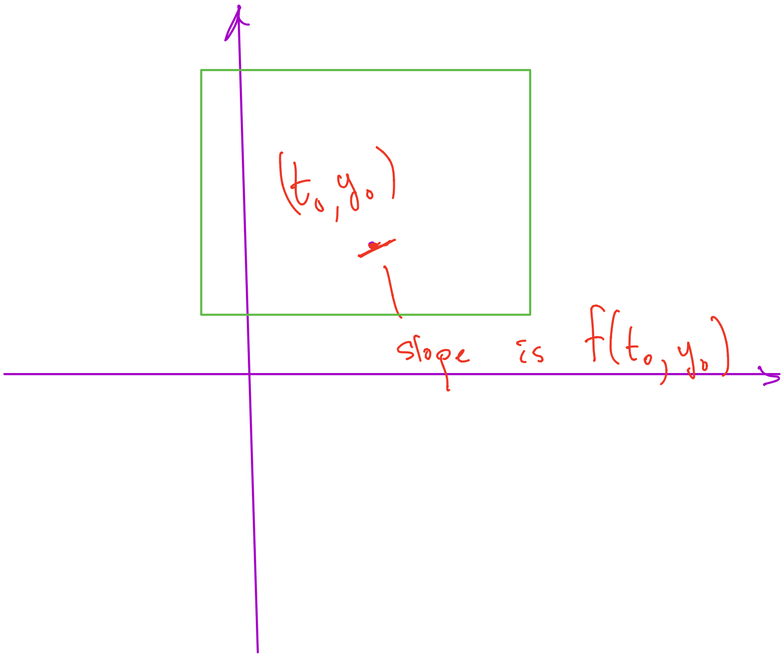
$$y' = \underbrace{f(t, y)}$$ Normal form

y' solved-for

terms on other side don't have y'

$y' = f(t, y)$ normal form

$y(t_0) = y_0$



$y_0 + \Delta t\, f(t_0, y_0)$