$$y'' - 2y' + y = 0 \quad \rightarrow \quad \lambda^2 - 2\lambda + 1 = 0$$

$$(\lambda - 1)^2 = 0$$

repeated char. val. $\lambda = 1$

$$\Rightarrow \underline{e^t}, \quad te^t \qquad \text{(basis of solns.)}$$

$$y_h = c_1 e^t + c_2 t e^t$$

Propose $y_p(t) = A \sin(t) e^{2t} + B \cos(t) e^{2t}$

$$y_p' = A \cos(t) e^{2t} + 2A \sin(t) e^{2t} - B \sin(t) e^{2t} + 2B \cos(t) e^{2t}$$

$$= e^{2t} \cos t (A + 2B) + e^{2t} \sin t (2A - B)$$

$$y_p'' = 2e^{2t} \cos t (A+2B) - e^{2t} \sin t (A+2B) + 2e^{2t} \sin t (2A-B) + e^{2t} \cos t (2A-B)$$

$$= e^{2t} \cos t \left[2(A+2B) + 2A - B \right] + e^{2t} \sin t \left[2(2A-B) - (A+2B) \right]$$

$$= e^{2t} \cos t (4A + 3B) + e^{2t} \sin t (3A - 4B)$$

Orig. nonhomog. Linear DE:

target

$$2 \sin(t) e^{2t} = y'' - 2y' + y \qquad \text{(insert } y_p, y_p', y_p'' \text{)}$$

$$= e^{2t} \cos t (4A + 3B) + e^{2t} \sin t (3A - 4B)$$

$$- 2 \left[e^{2t} \cos t (A + 2B) + e^{2t} \sin t (2A - B) \right]$$

$$+ A \sin(t) e^{2t} + B \cos(t) e^{2t}$$

$$= e^{2t}\cos t\left[(4A+3B) - 2(A+2B) + B\right]$$
$$+ e^{2t}\sin t\left[(3A-4B) - 2(2A-B) + A\right]$$

$$= e^{2t}\cos t\,(2A + 0B) + e^{2t}\sin t\,(0A - 2B)$$

Target

$$2e^{2t}\sin t + 0e^{2t}\cos t$$

Equate coeffs for various terms

term types	Ly - side		target side		
$e^{2t}\cos t$	$2A$	$=$	0	$\Rightarrow$	$A = 0$
$e^{2t}\sin t$	$-2B$	$=$	2	$\Rightarrow$	$B = -1$

$$y_p = -\cos(t)\,e^{2t}$$

general sol^n:

$$y_p + y_c = c_1 e^t + c_2 t e^t - e^{2t}\cos t.$$

$$y'' - 9y = e^{3t}$$

Start w/ homog. $y'' - 9y = 0 \rightarrow \lambda^2 - 9 = 0$

$$(\lambda - 3)(\lambda + 3) = 0$$

char. vals: $\lambda = \pm 3$

$\longrightarrow$ basis sol^{ns}. e^{3t}, e^{-3t}

$$y_h = c_1 e^{3t} + c_2 e^{-3t}$$

Target fn. (nonhomog. term) : $f(t) = e^{3t}$

$(0^{th}\text{-deg. poly.})(\text{exponential})$

Natural to propose

$$y_p(t) = \underbrace{A}_{0^{th}\text{ deg. poly.}} \underbrace{t\, e^{3t}}_{\text{exponential}}$$

— bad, since special instance of y_h.

So, modify by adding a factor t.

Process for determining A now proceeds as in previous examples.

$$y'' - 2y' + y = (7t-1)e^t$$

char. eqn. $\lambda^2 - 2\lambda + 1 = 0 \implies \lambda = 1$ (repeated)

$$y_h(t) = c_1 e^t + c_2 t e^t = (c_1 + c_2 t)e^t$$

target $(7t-1)e^t$ $(1^{st}\text{ deg. poly.})(\text{exponential})$

Propose

$$y_p(t) = (At + B)e^t \cdot t^2$$

Ex.

$$y'' + 2y' + 2y = e^{-2t} \cos t$$

char. eqn. $\lambda^2 + 2\lambda + 2 = 0$

$$(\lambda^2 + 2\lambda + 1) = -1$$

$$(\lambda + 1)^2 = -1$$

$$\lambda + 1 = \pm i$$

$$\lambda = -1 \pm i \qquad (\alpha = -1, \beta = 1)$$

homog. $y_h = c_1 e^{-t} \cos t + c_2 e^{-t} \sin t$

Propose

$$y_p(t) = A e^{-2t} \cos t + B e^{-2t} \sin t$$