Start w/ $f(t)$, domain "includes" $(0, \infty)$

$$\mathcal{L}\{f(t)\}(s) = \int_0^\infty e^{-st} f(t)\, dt$$

Started (Monday) a table of "time-domain" fns. and their Laplace transforms. Thus far, can handle polynomials.

Ex.] exponential fn. $f(t) = e^{at}$

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} \cdot e^{at}\, dt = \int_0^\infty e^{-(s-a)t}\, dt$$

$$= \frac{-1}{s-a} e^{-(s-a)t} \Big|_0^\infty$$

under assumption $s-a>0$

$$= \frac{-1}{s-a}(0-1) = \frac{1}{s-a}$$

Ex.] $f(t) = \sin(at)$

$$\mathcal{L}\{\sin(at)\} = \int_0^\infty e^{-st} \sin(at)\, dt$$

$u = e^{-st}$ | $du = -se^{-st}\, dt$

$dv = \sin(at)\, dt$ | $v = \frac{-1}{a}\cos(at)$

Aside

$$\int_0^A \underbrace{e^{-st}}_{u} \underbrace{\sin(at)\, dt}_{dv} = \underbrace{-\frac{1}{a} e^{-st} \cos(at) \Big|_0^A}_{u \cdot v} - \underbrace{\int_0^A -\frac{1}{a}(-s) e^{-st} \cos(at)\, dt}_{v \cdot du}$$

$$= -\frac{1}{a} e^{-sA} \cos(Aa) - -\frac{1}{a} - \frac{s}{a} \int_0^A e^{-st} \cos(at)\, dt$$

$u = e^{-st}$ | $du = -se^{-st}\, dt$

$dv = \cos(at)\, dt$ | $v = \frac{1}{a}\sin(at)$

new

$uv - \int v\,du$

$$= \frac{1}{a}\left(1 - e^{-sA}\cos(Aa)\right) - \frac{s}{a}\left(\frac{1}{a}e^{-st}\sin(at)\Big|_0^A - \int_0^A -\frac{s}{a}e^{-st}\sin(at)\,dt\right)$$

$$= \frac{1}{a}\left(1 - e^{-sA}\cos(Aa)\right) - \frac{s}{a}\left(\frac{1}{a}e^{-sA}\sin(Aa) + \frac{s}{a}\int_0^A e^{-st}\sin(at)\,dt\right)$$

Now have

$$\underbrace{\int_0^A e^{-st}\sin(at)\,dt}_{I} = \frac{1}{a} - \frac{1}{a}e^{-sA}\cos(Aa) - \frac{s}{a^2}e^{-sA}\sin(Aa) - \underbrace{\frac{s^2}{a^2}\int_0^A e^{-st}\sin(at)\,dt}_{-\frac{s^2}{a^2}I}$$

$$\left(1 + \frac{s^2}{a^2}\right)\int_0^A e^{-st}\sin(at)\,dt = \frac{1}{a} - \frac{1}{a}e^{-sA}\cos(Aa) - \frac{s}{a^2}e^{-sA}\sin(Aa)$$

$$= \frac{a^2}{a^2} + \frac{s^2}{a^2}$$

$$= \frac{a^2+s^2}{a^2}$$

Dividing by $\frac{a^2+s^2}{a^2}$, get

$$\int_0^A e^{-st}\sin(at)\,dt = \frac{a^2}{a^2+s^2}\left[\frac{1}{a} - \frac{1}{a}e^{-sA}\cos(Aa) - \frac{s}{a^2}e^{-sA}\sin(Aa)\right]$$

$$\mathcal{L}\{\sin(at)\} = \lim_{A\to\infty}\left(\text{this} \right) = \frac{1}{a}\cdot\frac{a^2}{s^2+a^2} = \frac{a}{s^2+a^2}$$

Alternative: simultaneously do $\sin(at)$, $\cos(at)$ thru Euler's formula

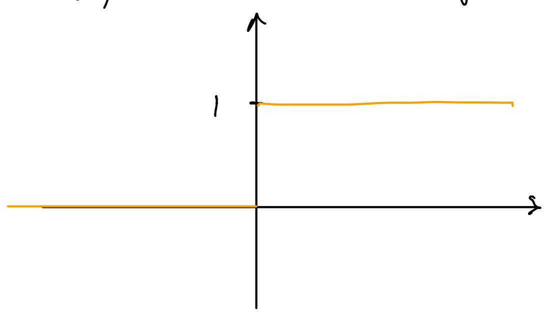
$$e^{iat} = \cos(at) + i\sin(at)$$

$$\mathcal{L}\{\cos(at) + i\sin(at)\} = \mathcal{L}\{e^{iat}\} = \frac{1}{s-ia}\cdot\frac{s+ia}{s+ia}$$

$$= \frac{s+ia}{s^2 - ias + ias - i^2a^2} = \frac{s+ia}{s^2+a^2}$$

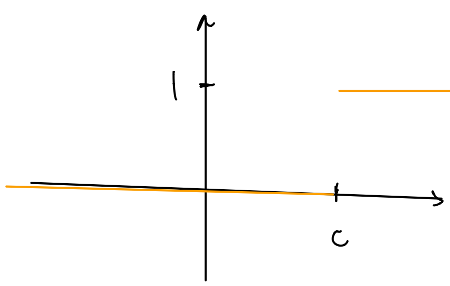
$$= \frac{s}{s^2+a^2} + i\,\frac{a}{s^2+a^2}$$

New (?) function: Heavyside unit step fn., $H(t) = U(t)$
$= u_0(t)$



$$= \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

Related:



$$H(t-c) = U(t-c)$$
$$= u_c(t)$$

time side $f(t)$	freq. side $\mathcal{L}\{f(t)\} = F(s)$
1	$\frac{1}{s}$, $s > 0$
t^n, $n = 1, 2, 3, \ldots$	$n!/s^{n+1}$, $s > 0$
e^{at}	$\frac{1}{s-a}$, $s - a > 0$
$\sin(at)$	$\frac{a}{s^2 + a^2}$, $s > 0$
$\cos(at)$	$\frac{s}{s^2 + a^2}$, $s > 0$