Math 251, Mon 30-Aug-2021 -- Mon 30-Aug-2021
Discrete Mathematics
Fall 2020

Monday, August 30th 2021

Wk 1, Mo
Topic:: Propositions
Read:: Rosen 1.1
HW:: PS01 due Mon.

Propositions

Identify those statements which are propositions (P), and those which are not (N).

1. P The house at 351 Riverside is burning.
2. N $2x + 5 = 11$.
3. P The cat is sleeping and the toddler is reaching for her tail.
4. N Put that down!
5. P Parking on the south side of the street is allowed only on even-numbered dates.
6. N This sentence is ~~a lie~~. not true.

Compound Propositions

All three of the statements

"atoms"

p: I write programs in C++.
q: I write programs in Python.
r: I write programs in C++ or in Python. p or q

are propositions. Propositions *p* and *q* seem more *atomic* than Proposition *r*; indeed, *r* is built out of these simpler ones, and is equivalent to *p* `OR` *q*. The making of a **compound proposition** out of two simpler ones joined by the word `OR` is called a **disjunction**.

The `OR` is a logical operator. There are others:

Name	Keyword	Symbol	Priority
negation	NOT	$\neg$	1
disjunction	OR	$\vee$	2
conjunction	AND	$\wedge$	2
	NAND?		
	NOR?		
exclusive or	XOR	$\oplus$	
conditional	IF . . . THEN . . .	$\rightarrow$	3
biconditional	IF AND ONLY IF	$\leftrightarrow$	3

For "order of operations":
- 1 before 2 before 3
- left-to-right when on the same priority level
- parentheses override these rules

Truth tables

p	q	$p \vee q$
F	F	F
F	T	T
T	F	T
T	T	T

$\vee$ = or (disjunction)

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

$\wedge$ = and (conjuction)

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

$p \to q$ read in English (implication, conditional)

"p implies q" "p is sufficient for q" "q unless $\neg p$"

"q if p" "q is necessary for p"

"if p then q" "p only if q"

p	q	$p \oplus q$
F	F	F
F	T	T
T	F	T
T	T	F

(exclusive or) XOR

p	q	$q \to p$
F	F	T
F	T	F
T	F	T
T	T	T

Truth table for $q \to p$ follows from that for $p \to q$

p	q	$p \leftrightarrow q$
F	F	T
F	T	F
T	F	F
T	T	T

(biconditional)

$p \leftrightarrow q$ same as $\neg(p \oplus q)$

"logically equivalent"

Translating to symbols

Define propositional variables p, q, (r, ...) and rewrite

1. Jenn is healthy, wealthy, but not wise.

   $p \wedge q \wedge \neg r$

   p: Jenn is healthy
   q: Jenn is wealthy
   r: Jenn is wise

2. John is neither healthy, wealthy, nor wise.

3. In order to rain, it must be cloudy.

4. I eat only when I am hungry.

5. It is not true that I am old and gray.