Math 251, Wed 1-Sep-2021 -- Wed 1-Sep-2021
Discrete Mathematics
Fall 2020

Wednesday, September 1st 2021

Wk 1, We
Topic:: Propositional logic
Note[[Worksheet
Read:: Rosen 1.1-1.2
HW((WW Propositions due Tues.

To discuss:
- satisfiability, knights & knaves problems
  A knight always tells the truth

  A knave always t~~ells the trut~~h lies.

  Example:
  1. Person A says B is a knight.
     Person B says A is a knave.
     unsatisfiable

  2. Person A says B is a knight.
     Person B says "we two are of opposite types."

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- related conditions to p -> q: inverse, converse, contrapositive
   example:  p: it rains,   q: it pours
   different ways to say, in English,  p -> q  and variants
- tautology, logical equivalence
- logical equivalence
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Warmup

Suppose we have atoms

p: You may take MATH 251. q: You may take MATH 171.

How does one translate into English

1. $p \vee q$?
2. $p \oplus q$?

Variants of conditional statements

Suppose we have atoms

p: It rains. q: It pours.

Give several English translations of $q \to p$.

If it pours, then it rains.

Pouring is sufficient for raining.

Raining is necessary for pouring.

Also of the converse $p \to q$

If it rains, it pours.

Also of the contrapositive $\neg p \to \neg q$

If it does not rain, then it does not pour.

Also of the inverse ~~[illegible]~~ $\neg q \to \neg p$

If it doesn't pour, it doesn't rain.

Tautologies and contradictions

A compound proposition formed from propositional variables p, q, etc. which is True regardless of the values of these variables, is called a **tautology**. If the negation of a compound proposition is a tautology, then the proposition itself is called a **contradiction**.

A very simple example of a tautology is : $p \vee \neg p$

A very simple example of a contradiction is : $p \wedge \neg p$

Returning to biconditionals

Recall $p \leftrightarrow q$ has truth table as give at left. Fill in the truth values missing for the table on the right.

p	q	$p \leftrightarrow q$
F	F	T
F	T	F
T	F	F
T	T	T

p	q	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	T	T	T

$$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$$

Similarly,

- compare truth of $p \to q$ with that of $\neg p \vee q$

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

p	q	$\neg p$		$\neg p \vee q$
F	F	T		T
F	T	T		T
T	F	F		F
T	T	F		T

- compare truth of $\neg(p \vee q)$ with that of $\neg p \wedge \neg q$

p	q	$\neg(p \vee q)$
F	F	T
F	T	F
T	F	F
T	T	F

p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	F	F	F

$$\neg(p \vee q) \equiv \neg p \wedge \neg q \quad \text{(DeMorgan's Law)}$$

- compare truth of $p \to q$ with that of $\neg q \to \neg p$ (contrapositive)

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

p	q	$\neg q$	$\neg p$	$\neg q \to \neg p$
F	F	T	T	T
F	T	F	T	T
T	F	T	F	F
T	T	F	F	T

$p \to q \equiv \neg q \to \neg p$

$P \equiv Q$ precisely when $P \leftrightarrow Q$ is a tautology.

Logical equivalence

We say two compound propositions P, Q are **logically equivalent**, written as $P \equiv Q$, , pecisely in the case that $P \leftrightarrow Q$ is a tautology.

From our work above, we have demonstrated three logical equivalences:

Some other logical equivalences (see Tables 6–8, pp. 27–28 for a more complete list):

- DeMorgan's Laws:
  $\neg(p \vee q) \equiv \neg p \wedge \neg q$
  $\neg(p \wedge q) \equiv \neg p \vee \neg q$
- Identity Laws:
  $p \wedge T \equiv p$
  $p \vee F \equiv p$
- Associative Laws:
  $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- Commutative Laws:
  $p \wedge q \equiv q \wedge p$
  $p \vee q \equiv q \vee p$
- Distributive Laws:
  $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Try simplifying the logical expression:

$$(p \wedge q) \vee (p \wedge \neg q) \equiv p \wedge (q \vee \neg q) \equiv p \wedge T \equiv p$$