

Math 251, Fri 3-Sep-2021 -- Fri 3-Sep-2021
 Discrete Mathematics
 Fall 2021

 Friday, September 03rd 2021

Due:: PS01

 Friday, September 3rd 2021

Wk 1, Fr
 Topic:: Satisfiability; Disjunctive normal form
 Read:: Rosen 1.2
 HW:: PS02 due Fri.

Warmup:

Use DeMorgan's laws to write the negation of

p : food is good q : service is excellent

(a) The food is good or the service is excellent.

$\neg(p \vee q) = \neg p \wedge \neg q$, or "the food is not good and the service isn't excellent"

(b) The food is good and the service is excellent.

$\neg(p \wedge q) =$ "the food is not good or the service is not excellent."

(c) The food is good and service excellent, or the price is high. r : price is high

The food is not or the service is not good and the price is not high.

Find a simpler proposition that is logically equivalent to

(a) not (p or (not p and q))

(a) $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg(\neg p \wedge q) \equiv \neg p \wedge (p \vee \neg q) \equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$

(b) not q and (p -> q) -> not p

$\equiv F \vee (\neg p \wedge \neg q) \equiv \neg p \wedge \neg q \equiv \boxed{\neg(p \vee q)}$

(b) $\neg q \wedge (p \rightarrow q) \rightarrow \neg p \equiv \neg q \wedge (q \vee \neg p) \equiv (\neg q \wedge q) \vee (\neg q \wedge \neg p) \equiv F \vee (\neg q \wedge \neg p)$

Meaning When:

- multiple conjunctions are strung together

$\equiv \neg q \wedge \neg p \equiv \boxed{\neg(q \vee p)}$

- multiple disjunctions are strung together

Satisfiability and consistency

A proposition is **satisfiable** if there is some set of truth values assignable to its propositional variables (atoms) under which the proposition is true. In other words, the proposition is *not* a contradiction.

Examples:

1. $\neg p$

2. $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$

is satisfiable

$\left. \begin{matrix} p \vee \neg q \\ q \vee \neg r \\ r \vee \neg p \end{matrix} \right\}$ are consistent

p	q	r	$\neg p$	$\neg q$	$\neg r$	$p \vee \neg q$	$q \vee \neg r$	$r \vee \neg p$	full expr.
F	F	F	T	T	T	T	T	T	T
F	F	T	T	T	F	T	F	T	F
F	T	F	T	F	T	F	T	T	F
F	T	T	T	F	F	F	T	T	F
T	F	F	F	T	T	T	T	F	F
T	F	T	F	T	F	T	F	T	F
T	T	F	F	F	T	T	T	F	F
T	T	T	F	F	F	T	T	T	T

3. $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$

p	q	r
F	F	F
F	F	T
F	T	F
F	T	T
T	F	F
T	F	T
T	T	F
T	T	T

4. $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \wedge (p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$

p	q	r	
F	F	F	
F	F	T	
F	T	F	
F	T	T	
T	F	F	
T	F	T	
T	T	F	
T	T	T	

5. Are these propositions **consistent** (simultaneously satisfiable)?

- You have a virus and you use the #CampusClear app.
- If you have a sore throat, the app says "don't come to class."
- You do not come to class when the app says not to.
- You have a sore throat only if you have a virus.
- You come to class.