Math 251, Wed 8-Sep-2021 -- Wed 8-Sep-2021
Discrete Mathematics
Fall 2021

Wednesday, September 08th 2021

Wk 2, We
Topic:: Predicate and quantifiers
Read:: Rosen 1.4
HW((WW PredicatesAndQuantifiers due Tues.
HW:: Quiz Ch. 1 ends Mon.

Administrative:
 - take attendance
 - mention quiz

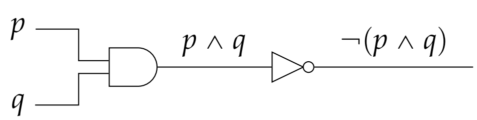
Disjunctive Normal Form

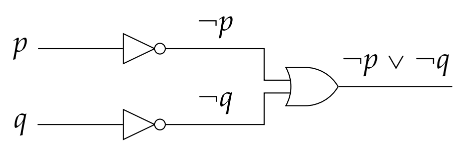
A compound proposition is in **disjunctive normal form** (DNF) if

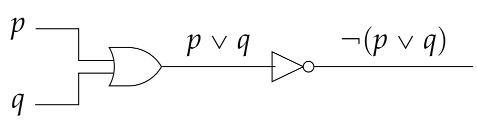
- negations occur only on the atomic propositions.
- conjunctions occur only on inputs containing no disjunctions.
- there are no operations besides negation, conjunction and disjunction.

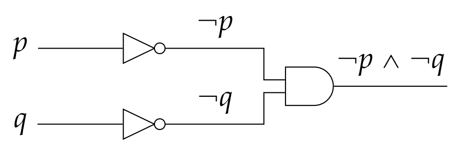
Several (tauto)logical equivalences can be used to re-express compound propositions in DNF.

- $p \to q \;\equiv\; \neg p \vee q$, to eliminate implications.
  Note how this provides direction for removing biconditionals, too.
- $p \oplus q \;\equiv\; (p \vee q) \wedge (\neg p \vee \neg q)$, to eliminate EXCLUSIVE ORs.
- $\neg(p \vee q) \;\equiv\; \neg p \wedge \neg q$, and $\neg(p \wedge q) \;\equiv\; \neg p \vee \neg q$, to move negation inside of conjunction/disjunction.

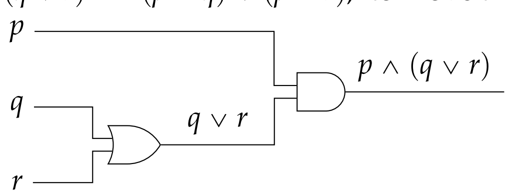


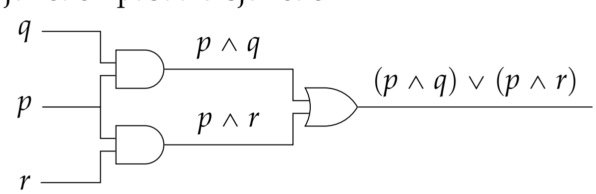






- $p \wedge (q \vee r) \;\equiv\; (p \wedge q) \vee (p \wedge r)$, to move a conjunction past a disjunction.





Exercise: Put the compound proposition $(p \to (q \wedge r)) \vee \neg(p \vee \neg(r \vee s))$ into DNF.
Answer: $(\neg p \vee (q \wedge r)) \vee ((\neg p \wedge r) \vee (\neg p \wedge s))$.

Conclusion: The three operators: $\neg$, $\wedge$, and $\vee$ are **functionally complete**.

Predicates

A **predicate**, or **propositional function**, is a statement involving at least one variable such that, when all variables are either

- assigned a value, or
- **bound** by a quantifier,

the result is a proposition. The **domain**, or **universe of discourse**, for each variable must be clear.

Examples

- $P(x)$: "x is a city in Michigan" with
  domain: place names.

  P(Philadelphia) is false, P(Detroit) is true

- $C(x, y)$: "$y = x^2 - 1$"
  domain: coordinate pairs (x, y) where both x, y are real numbers

  C(1, 0) is true

- $A(x, y)$: "The word x contains the letter y"
  domain: (x, y) consists of a word x and a letter y

  A(Mississippi, i) is true

Quantifiers. We indicate the

- **universal quantifier**: the symbol $\forall$ is read aloud as "for all" or "for every."
- **existential quantifier**: the symbol $\exists$ is read aloud as "there exists" or "some."
- **uniqueness quantifier**: the symbol $\exists!$ is read aloud as "there exists a unique" or "there is precisely one."

Examples:

1. $P(x)$: "x is mortal"
   domain: human beings C.S. Lewis says F
   translate $\forall x\, P(x)$ All human beings are mortal.
2. If D is the set of numbers $D = \{1, 2, 3, 4, 5\}$, is this statement true?
   $\forall x \in D\ (x^2 \geqslant x)$ evaluates T
3. If x, y are from the domain $\mathbb{R}$, is the statement true?
   $\forall x\, \forall y\ (xy = yx)$ evaluates T

nested quantifiers

4. Say the domain for x is all real numbers. Translate
   $\exists x\ (x^2 = 2)$ True "There is an x in $\mathbb{R}$ satisfying $x^2 = 2$."
5. Translate
   $\forall x < 0\ (x^2 > 2)$
6. Interpret the statement: $\forall x((x \neq 0) \to \exists! y(xy = 1))$
7. Say the domains for a_0, a_1, a_2, a_3, x are $\mathbb{R}$. Interpret
   $\forall a_0 \forall a_1$ $\forall a_2 \forall a_3\ ((a_0 \neq 0) \to \exists x(a_0x^3 + a_1x^2 + a_2x + a_3 = 0))$.
8. Interpret $\exists! x$(x is omniscient, omnipresent and omnipotent)

Notes:

- The uniqueness quantifier is convenient, but extraneous.
- Quantifiers take precedence over logical operators. Thus

$$\forall x P(x) \wedge Q(x) \quad \text{means} \quad (\forall x P(x)) \wedge Q(x), \quad \text{not} \quad \forall x(P(x) \wedge Q(x)).$$

  The latter is logically equivalent to $\forall x P(x) \wedge \forall x Q(x)$.
- A variable in a predicate that has no value or quantifier is **free**.
- **Negating quantifiers**. The negation of $\forall x\, P(x)$ is $\exists x\ \neg P(x)$

  The negation of $\exists x\, P(x)$ is $\forall x\ \neg P(x)$

$\forall x\ P(x,y)$ is not a proposition, as y is free.

Practice

Websites:

http://scofield.site/courses/m251/worksheets/sWars.txt

https://www.tutorialspoint.com/execute_prolog_online.php

1. Write queries for
   (a) whether luke is a child of leia.
   (b) all children of leia.
   (c) all sons of leia.
   (d) all uncles of jacen.
   (e) all grandchildren of anakin
   (f) all names of ”force-sensitive” characters (whether sith or jedi)
   (g) all names of characters who are both sith and jedi
2. Write rules for
   (a) mother(X), so that the mother of X is sought/found
   (b) nephew(X)
   (c) isForceSensitive(X)
   (d) isForceSensitive(X)
   (e) grandfather(X)
3. Add information to the knowledge base so that there is a person named owen who appears in response to the query `uncle(luke)`.