Math 251, Fri 10-Sep-2021 -- Fri 10-Sep-2021
Discrete Mathematics
Fall 2021

Friday, September 10th 2021

Wk 2, Fr
Topic:: Nested quantifiers
Read:: Rosen 1.5
[redacted].

Nested quantifiers

If the statement is true, indicate that. If not, write a "nearby" statement to fix it.

- $\forall x\, \forall y\, (x + y = 0)$ $\quad \forall x\; \exists y\; (x + y = 0)$

- $\exists x\, \forall y\, ((x > 0) \to (y^2 = x))$ $\quad \forall y\; \exists x\; ((y^2 = x) \wedge (x \geq 0))$

$$\forall x\; (x > 0 \longrightarrow \exists y\; (y^2 = x))$$

Use quantifiers and symbols to codify the English statement.

- There is a supervisor who oversees every process in the factory $\quad P(x,y)$: (person) x oversees (process) y

$$\exists x\; \forall y\; P(x,y)$$

- Every process in this factory is overseen by some person.

$$\forall y\; \exists x\; P(x,y)$$

- There is a positive integer that is smallest (i.e., at least as small as any other).

$$\exists x\; \forall y\; (y \geq x)$$

- There is a real number that has no reciprocal (multiplicative inverse).

$$\exists x \in \mathbb{R}\; \forall y \in \mathbb{R}\; (xy \neq 1)$$

- There is no smallest real number. [with negation symbol? without one?]

$$\forall x \in \mathbb{R}\; \exists y \in \mathbb{R}\; (y < x)$$

- Any secret any person knows can be revealed to the right person.

$K(x, s)$: (person) x knows (secret) s

$R(x, y, s)$: (person) x reveals (secret) s to (person) y

$$\forall s\; \forall x\; (K(x,s) \longrightarrow \exists! y\; R(x,y,s))$$

Negating nested quantifiers

- $\neg\forall x\,\exists y\,P(x,y) \;\equiv\; \exists x\,\forall y\,\neg P(x,y)$
- $\neg\exists x\,\forall y\,P(x,y) \;\equiv\; \forall x\,\exists y\,\neg P(x,y)$
- $\neg\forall x\,\forall y\,P(x,y) \;\equiv\; \exists x\,\exists y\,\neg P(x,y)$
- $\neg\exists x\,\exists y\,P(x,y) \;\equiv\;$ $\forall x\,\forall y\;\neg P(x,y)$
- $\neg\forall x\,\exists y\,\forall z\,P(x,y,z) \;\equiv\;$ $\exists x\,\forall y\,\exists z\;\neg P(x,y,z)$

Rewrite the statement in as simple an English statement as possible. Then state the negation of that statement.

- $\forall$ colors C, $\exists$ an animal A such that A is colored C.

  S: No matter the color, some animal is that color.

  ¬S: There is some color that no animal has.
- $\exists$ a book b such that $\forall$ people p, p has read b.

  S: There is a book everyone has read.

  ¬S: No matter the book, someone has not read it.
- $\forall$ odd integers n, $\exists$ an integer k such that $n = 2k+1$.

  S: Every odd integer is one greater than twice some integer.

  ¬S: There exists an odd integer that doesn't equal $2k+1$ for any integer k.
- $\exists$ real x such that for all real y, $x+y=0$.

  S: Some real number x serves simultaneously as additive inverse to all reals y.

  ¬S: No matter the choice of x, some real y exists so that $x+y \neq 0$.