Math 251, Mon 13-Sep-2021 -- Mon 13-Sep-2021
Discrete Mathematics
Fall 2021

Monday, September 13th 2021

Wk 3, We
Topic:: Sets
Read:: Rosen 2.1
Due:: Quiz Ch. 1 ends at 10 pm

Sets

Sets

- collections of objects

  We'll avoid descriptions generating paradoxes

- ways of describing
  - enumeration

    {Su, Tu, We, Sa, Fr, Mo, Th} $\{\ldots, -4, -2, 0, 2, \ldots\}$

    {1, 2, 3, ..., 100}

    {-7, -6, -5, ..., 0, 1, ...}
  - set builder notation

    $\mathbb{Q} = \{ m/n \mid m, n \in \mathbb{Z},\ n \neq 0 \}$

    $\{ x \mid x^2 + 3x + 2 = 0 \} = \{-1, -2\}$
  - intervals of numbers

    $(-\infty, \infty)$

    $[2, 5) = \{ x \mid x \geq 2 \ \wedge \ x < 5 \}$
  - words/symbols

    $\mathbb{R}$ - real nos

    $\mathbb{N}$ - natural nos

    $\mathbb{Z}$ - integers

    $\mathbb{Z}^+$ - positive integers

    $\mathbb{Q}$ - rational numbers

    $\mathbb{C}$

- items in a set are called **elements** of that set
  Some notions and how to express them
  - "is/is not an element of"

$7 \in \mathbb{N}$

$-1 \notin \mathbb{N}$

Empty set

$\emptyset$ $\{\}$

By convention, $\emptyset$ is a subset of every set.

  - "is/is not a subset of"

A, B sets, say $A \subseteq B \leftrightarrow ((x \in A) \rightarrow (x \in B))$

Say $A = \{0, 1, 2\}$, $B = \{0, 1, 2, \ldots 5\}$

True: $1 \in A$, $1 \in B$ False: $\{1\} \in A$

True: $\{1\} \subseteq A$, $A \subseteq B$

  - "equality of sets"

A, B sets are equal precisely when the contain the same elements.

$\{1, 3, 2\} = \{1, 2, 3\}$

$\{1, 1, 2, 1, 3, 1\} = \{1, 2, 3\}$

Fact: $A = B \leftrightarrow (A \subseteq B) \wedge (B \subseteq A)$

- can be empty, finite or infinite
  cardinality = count of elements in a set

If $A = \{2, 7, 10\}$, then $|A| = 3$

Sets built from other sets

- the power set $\mathcal{P}(A)$ of a set A
  use of bitstrings to describe subsets of a finite set A

Given $A = \{0, 1\}$

$\mathcal{P}(A) = \{\emptyset, \{0\}, \{1\}, \{0, 1\}\}$

0000 ↑ 0-element subsets

1000, 0100, 0010, 0001 ↑ 1-element subsets

1100, 1010, 1001, 0110, 0101, 0011 ↑ 2-element subsets

1110, 1101, 1011, 0111 ↑ 3 element subsets

1111 ↑ 4-element subset

For a given set A, $\mathcal{P}(A) = \{B \mid B \subseteq A\}$

$\mathcal{P}(\mathcal{P}(A)) = \{\emptyset\}, \{\{0\}\}, \{\{1\}\}, \{\{0,1\}\}, \{\emptyset, \{0\}\}, \{1\}, \{0,1\}\},$
$\{\emptyset, \{0\}\}, \{\emptyset, \{1\}\}, \{\emptyset, \{0,1\}\}, \{\{0\}, \{1\}\}, \{\{0\}, \{0,1\}\},$
$\{\{1\}, \{0,1\}\}, \{\}$, and 4 more

$|\emptyset| = 0$

$\mathcal{P}(\emptyset) = \{\{\}\}$

$\mathcal{P}(\mathcal{P}(\emptyset)) = \{\{\}, \{\{\}\}\}$

- Cartesian product $A \times B$ of two sets A, B

Do Wed.