

Math 251, Mon 13-Sep-2021 -- Mon 13-Sep-2021
Discrete Mathematics
Fall 2021

Monday, September 13th 2021

Wk 3, We

Topic:: Sets

Read:: Rosen 2.1

Due:: Quiz Ch. 1 ends at 10 pm

Sets

Sets

- collections of objects

We'll avoid descriptions generating paradoxes

- ways of describing

- enumeration

$\{\text{Su, Tu, We, Sa, Fr, Mo, Th}\}$

$\{\dots, -4, -2, 0, 2, \dots\}$

$\{1, 2, 3, \dots, 100\}$

$\{-7, -6, -5, \dots, 0, 1, \dots\}$

- set builder notation

$\mathbb{Q} = \{m/n \mid m, n \in \mathbb{Z}, n \neq 0\}$

$\{x \mid x^2 + 3x + 2 = 0\} = \{-1, -2\}$

- intervals of numbers

$(-\infty, \infty)$

$[2, 5) = \{x \mid x \geq 2 \wedge x < 5\}$

- words/symbols

\mathbb{R} - real nos

\mathbb{N} - natural nos

\mathbb{Z} - integers

\mathbb{Z}^+ - positive integers

\mathbb{Q} - rational numbers

\mathbb{C}

- items in a set are called **elements** of that set

Some notions and how to express them

- "is/is not an element of"

$$7 \in \mathbb{N}$$

$$-1 \notin \mathbb{N}$$

Empty set

$$\emptyset \quad \{\}$$

By convention, \emptyset is a subset of every set.

- "is/is not a subset of"

$$A, B \text{ sets, say } A \subseteq B \leftrightarrow ((x \in A) \rightarrow (x \in B))$$

$$\text{Say } A = \{0, 1, 2\}, B = \{0, 1, 2, \dots, 5\}$$

$$\text{True: } 1 \in A, 1 \in B \quad \text{False: } \{1\} \in A$$

$$\text{True: } \{1\} \subseteq A, A \subseteq B$$

- "equality of sets"

A, B sets are equal precisely when they contain the same elements.

$$\{1, 3, 2\} = \{1, 2, 3\}$$

$$\{1, 1, 2, 1, 3, 1\} = \{1, 2, 3\}$$

$$\text{Fact: } A = B \leftrightarrow (A \subseteq B) \wedge (B \subseteq A)$$

- can be empty, finite or infinite
cardinality = count of elements in a set

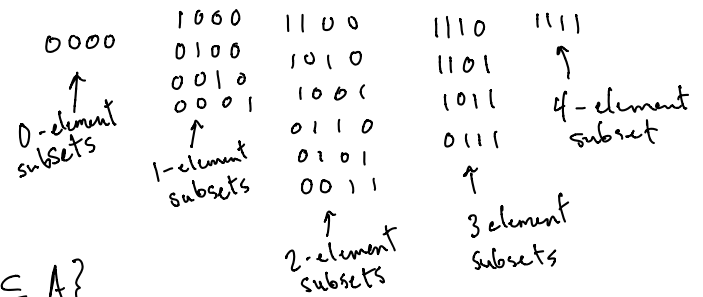
$$\text{If } A = \{2, 7, 10\}, \text{ then } |A| = 3$$

Sets built from other sets

- the power set $\mathcal{P}(A)$ of a set A
 use of bitstrings to describe subsets of a finite set A

Given $A = \{0, 1\}$

$\mathcal{P}(A) = \{\emptyset, \{0\}, \{1\}, \{0, 1\}\}$



For a given set A , $\mathcal{P}(A) = \{B \mid B \subseteq A\}$

$\mathcal{P}(\mathcal{P}(A)) = \{\emptyset, \{\{0\}\}, \{\{1\}\}, \{\{0, 1\}\}, \{\emptyset, \{0\}, \{1\}, \{0, 1\}\},$
 $\{\emptyset, \{0\}\}, \{\emptyset, \{1\}\}, \{\emptyset, \{0, 1\}\}, \{\{0\}, \{1\}\}, \{\{0\}, \{0, 1\}\},$
 $\{\{1\}, \{0, 1\}\}, \{\}, \text{ and 4 more}$

$|\emptyset| = 0$

$\mathcal{P}(\emptyset) = \{\{\}\}$

$\mathcal{P}(\mathcal{P}(\emptyset)) = \{\{\}, \{\{\}\}\}$

- Cartesian product $A \times B$ of two sets A, B

Do Wed.