Math 251, Wed 15-Sep-2021 -- Wed 15-Sep-2021
Discrete Mathematics
Fall 2021

Wednesday, September 15th 2021

Wk 3, We
Topic:: Set operations
HW:: PS04 due Wed.
Read:: Rosen 2.2

$\forall$, $\exists$, $\exists!$

$$\exists! x\ P(x) \equiv \exists x \left(P(x) \wedge \forall y \left(P(y) \to (y = x)\right)\right)$$

Cartesian products

Exactly two

$$\exists x \exists y \left((x \neq y) \wedge P(x) \wedge P(y) \wedge \forall z \left(P(z) \to (z = x) \vee (z = y)\right)\right)$$

Given sets A, B, define $A \times B = \{ \text{coord. pairs} \mid 1^{st} \text{ coord.} \in A,\ 2^{nd} \text{ coord} \in B \}$

Ex.) $A = \{1, 2, 3\}$, $B = \{a, b, c, d\}$

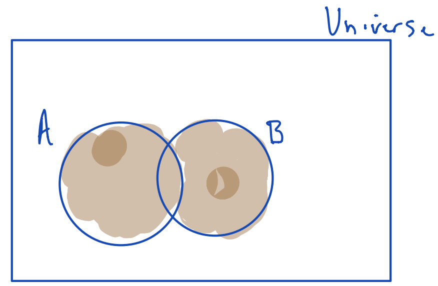
$(1, c) \in A \times B$ but $(c, 1) \notin A \times B$, though $(c, 1) \in B \times A$

$(1, c, d, 3) \in A \times B \times B \times A$

$\mathbb{R} \times \mathbb{R}$ (often written as $\mathbb{R}^2$)

Sets built from other sets

- Union of sets (two, more than two)



Given A, B, define $A \cup B = \{x \mid x \in A \vee x \in B\}$

membership table

A	B	$A \cup B$
0	0	0
0	1	1
1	0	1
1	1	1

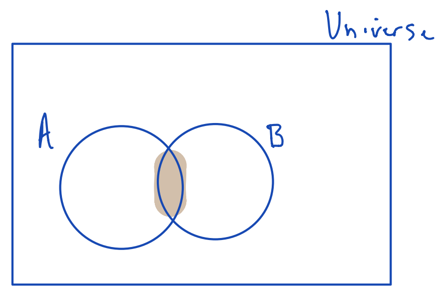
$A \cup B \cup C$

If I have infinitely many sets

$A_1, A_2, A_3, \ldots$

$$\bigcup_{n=1}^{\infty} A_n$$

- Intersection of sets (two, more than two)



$A \cap B = \{x \mid x \in A \wedge x \in B\}$

Say A and B are disjoint whenever $A \cap B = \emptyset$.

A, B disjoint

$$|A \cup B| = |A| + |B|$$

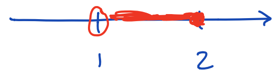
- Set subtraction and complementation
  - disjoint sets
  - breaking $A \cup B$ into a disjoint union
  - inclusion-exclusion principle
  - complement arises from set subtraction from a universal set

Define $A - B = \{x \mid x \in A \wedge x \notin B\}$

This allows us to break up

$$A \cup B = (A - B) \cup (A \cap B) \cup (B - A)$$

into disjoint "pieces" (subsets)

Example: Say $A_1 = (1, 2]$ 

$A_2 = (\frac{1}{2}, 2]$

generally, $A_n = (\frac{1}{n}, 2]$

Then $\bigcup_{n=1}^{\infty} A_n = (0, 2]$

For a given set A, define $\bar{A}$ (read as "complement of A")

$\bar{A} = \{x \mid x \text{ is in universe but } x \notin A\}$.

Ex.

$A = \{a, e, i, o, u\}$

$\rightarrow \bar{A} = \{\text{consonants}\}$

Aside: In Chapter 1 saw DeMorgan's Laws

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Guess:

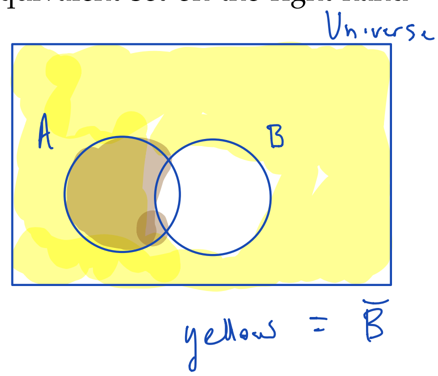
$\overline{A \cup B} = \bar{A} \cap \bar{B}$

Check it w/ membership table

A	B	$A \cup B$	$\overline{A \cup B}$	$\bar{A}$	$\bar{B}$	$\bar{A} \cap \bar{B}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

Identities (akin to logical equivalences in Chapter 1)

Using your intuition, Venn diagrams, etc., present a plausibly equivalent set on the right-hand side, then prove it (first to yourself).



1. $A - B =$ $A \cap \overline{B}$

2. $A \cup \overline{A} =$ universe

3. $A \cap A =$ A

4. $(A \cap B) \cap C =$ $A \cap (B \cap C)$

5. $(A \cap B) \cup C =$ $(A \cup C) \cap (B \cup C)$

6. $\overline{A \cup B} =$ $\overline{A} \cap \overline{B}$

7. $A \cup (A \cap B) =$

8. $\overline{A \cup (B \cap C)} =$

Methods for proving two sets are equal, i.e., $A = B$

- Show $A \subseteq B$ and $B \subseteq A$ (example: 6)

- Invoke set builder description, use logical equivalences (example: 6)

- Show that A and B have the same membership table (example: 8)

Set operations compared with bit operations