Math 251, Mon 20-Sep-2021 -- Mon 20-Sep-2021

Discrete Mathematics

Fall 2021

Monday, September 20th 2021

Wk 4, Mo

Topic:: Functions

Read:: Rosen 2.3

HW:: WW functions due Thurs.

[redacted].

(a) f(1101) = 3 f(100101)

(b) f(1101) = 3, f(100101) = 3

Warmup:

1. For a bit string S, let f(S) give
   a) the position of a 0-bit in S No
   b) the number of 1-bits in S Yes
   In either case, is f a function from the set of all bit strings to Z?

2. Given f(x) = 3x+1 and g(x) = sqrt(x).
   a) Is f a function from R to R?
   b) Is f surjective? ✓
   c) Is f injective? ✓
   f is bijective
   d) Does f have an inverse function? If so, what is it? $f^{-1}(x) = x/3 - 1/3$
   e) Is g(f(x)) a function from R to R? (mention partial functions)
   No

Some special functions

- characteristic functions on a set

  Given any set A

  $$\chi_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

- floor and ceiling functions
   1. arrange in ascending order: ceil(x), floor(x), x-1, x+1, x

   2. give an alternate expression for
      floor(-x)
      ceil(-x)
      floor(x + n), where n is an integer
      floor(2x)

- logarithms

Did not discuss in class - review it.

```
For b > 1, define log_b(x)
Properties of logs
  inverse to b^x
  log_b (xy) = log_b x + log_b y
  log_b (x/y) = log_b x - log_b y
  log_b (x^a) = a log_b x
  log_a x = log_b x / log_b a
```

"f is a fn. from <u><domain></u> to <u><codomain></u>."

Other phrases:

"f is a fn. of a real variable" ... $\longleftrightarrow$ domain $= \mathbb{R}$

"f is real-valued" $\longleftrightarrow$ codomain $= \mathbb{R}$

f is surjective precisely when: range(f) = codomain(f)

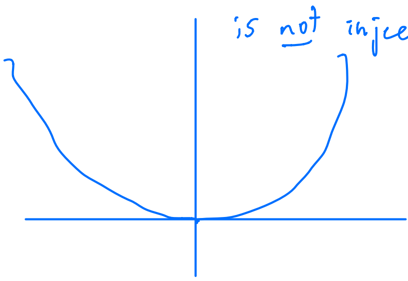
$\forall y \in$ codomain $\exists x \in$ domain $(f(x) = y)$.

f is injective precisely when: $\forall x_1 \in$ domain $\forall x_2 \in$ domain $(f(x_1) = f(x_2) \to x_1 = x_2)$.

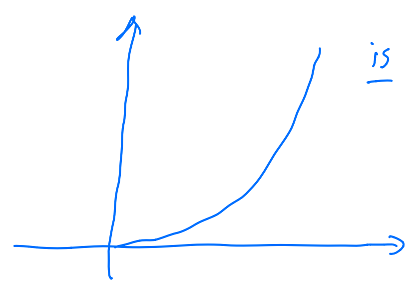
(i.e. cannot get the same y-value out of f from 2 different x-values.)

Ex.) $f(x) = x^2$

as a fn. from $\mathbb{R}$ to $\mathbb{R}$ is <u>not</u> injective



as a fn. from $[0, \infty)$ to $\mathbb{R}$ <u>is</u> injective



A bijective fn. is both surjective and injective. As a result, it has an inverse that is a function returning from B to A (if $f: A \to B$).

Inverse of $f(x) = 3x + 1$ $\qquad y = 3x + 1$

Solve for x: $\qquad y - 1 = 3x$

$$x = \tfrac{1}{3}y - 1/3$$

$$f^{-1}(x) = \boxed{x/3 - 1/3}$$

$$(g \circ f)(x) = g(f(x))$$

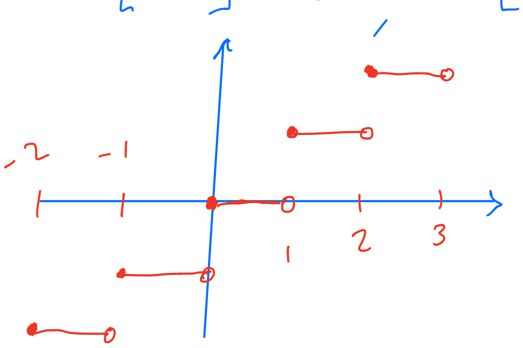
For example above

$$g(f(x)) = g(3x+1) = \sqrt{3x+1}$$

In Python $\quad \text{floor}(x)$ = largest integer not exceeding x

$$= \lfloor x \rfloor$$

Ex.] $\lfloor 2.1 \rfloor = 2$, $\quad \lfloor 2 \rfloor = 2$, $\quad \lfloor -3.9 \rfloor = -4$



Some special funcitons

Identity function. $\iota\colon A \to B$ requires $A \subseteq B$.

Indicator functions. Given a set $A \subseteq \mathbb{R}$, the indicator function on the set A is defined as

$$\chi_A(x) \;:=\; \begin{cases} 1, & \text{if } x \in A, \\ 0, & \text{if } x \notin A. \end{cases}$$

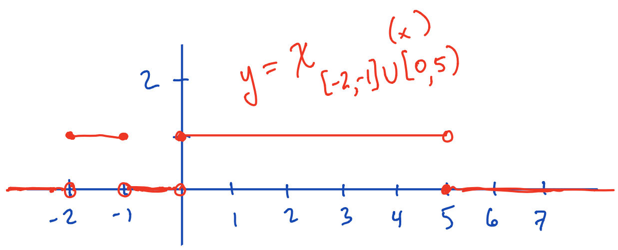
- evaluating an indicator function

$\chi_{[0,5)}(5) = 0$ but $\chi_{[0,5)}(\pi) = 1$.

- graph of an indicator function

Let $A = [-2,-1] \cup [0,5)$

Its graph is depicted here:



The floor/ceiling functions.

- how defined
- True or false?
  1. $\forall x \in \mathbb{R}\, \forall y \in \mathbb{R}\, (\lfloor x + y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor)$ False
  2. $\forall x \in \mathbb{R}\, \forall m \in \mathbb{Z}\, (\lfloor x + m \rfloor = \lfloor x \rfloor + m)$ True
  3. $\forall x \in \mathbb{R}\, (\lfloor -x \rfloor = -\lfloor x \rfloor)$
  4. $\forall x \in \mathbb{R}\, (\lfloor 2x \rfloor = \lfloor x \rfloor + \lfloor x + 0.5 \rfloor)$
  5. $\lfloor\,\rfloor\colon \mathbb{R} \to \mathbb{Z}$ is a bijection.