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Math 251, Wed 22-Sep-2021 -- Wed 22-Sep-2021
Discrete Mathematics
Fall 2021


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Wednesday, September 22nd 2021
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Due::   PS04 due at 11 pm

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Wk 4, We    Topic: Sequences
Topic:: Bijections
Topic:: Countability
HW:: Moodle Quiz Chs. 1-2 ends at Sat.

- Logarithms
   For b > 1, define log_b(x)
   Properties of logs
     inverse to b^x
     log_b (xy) = log_b x + log_b y
     log_b (x/y) = log_b x - log_b y
     log_b (x^a) = a log_b x
     log_a x = log_b x / log_b a
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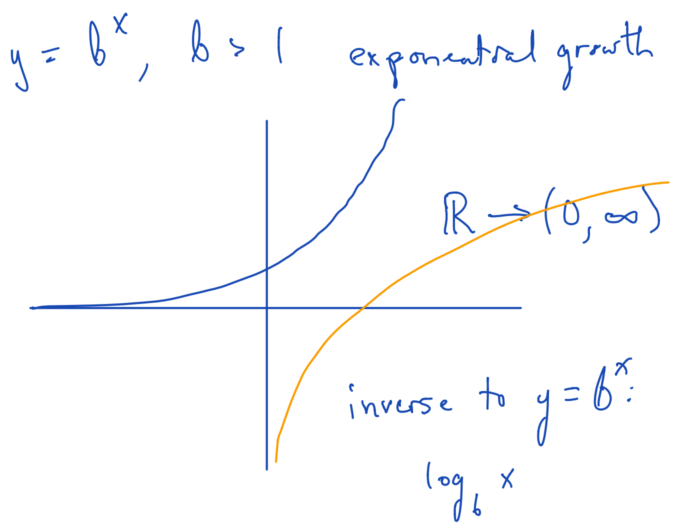


$\log_b b^x = x$

$b^{\log_b x} = x$

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Sequences
 - (partial) functions from N to R
 - notation
 - examples:
    Fibonacci
    constant
    arithmetic
    geometric

Arithmetic sequences:
 - each a_n = a_{n-1} + d,   where d is the common difference
    ==> a_n = a_{n-1} + d
        a_n = a_{n-2} + 2d
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        a_n = a_{n-3} + 3d
           ...
        a_n = a_0 + nd     (closed formula, requires a_0, d)
 - summing terms
    2 + 9 + 16 + 23 + ... + 100
    general: a_0 + a_1 + a_2 + ... + a_n = (n+1)(a_0 + a_n) / 2

Geometric sequences
 - each a_n = ra_{n-1},   where r is the common difference
    ==> a_n = a_{n-1} r
        a_n = a_{n-2} r^2
        a_n = a_{n-3} r^3
           ...
        a_n = a_0 r^n     (closed formula, requires a_0, r)

 - summing terms
    2 + 6 + 18 + 54 + ... + 4374   (last term is 2*3^7)
    general: a_0 + a_0r + a_0r^2 + ... + a_0 r^n = ...

Say you borrow 10K at 8% interest, compounded monthly.
  If at the end of each month you pay $250,
    how much will you owe after one month?


    how much will you owe after n months?
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Is it injective?

(a) $f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2 + 5$

No. So not invertible.

$$f^{-1}(\{5, 6, 9\}) = \{x \mid f(x) = 5 \lor f(x) = 6 \lor f(x) = 9\}$$
$$= \{0, 1, 2, -1, -2\}$$

(b) $f: [0, \infty) \to \mathbb{R}$ w/ $f(x) = x^2 + 5$ is injective.

Is it surjective

(a) $f: [0, \infty) \to \mathbb{R}$ w/ $f(x) = x^2 + 5$

No

(b) Can we simply change the codomain to make $f(x) = x^2 + 5$ bijective?

Yes. Make it $f: [0, \infty) \to [5, \infty)$

(c) $f: \mathbb{R} \to \mathbb{R}$ given $f(x) = 3x - 1$

Yes.

# Sequences

- functions $\mathbb{N} \to \mathbb{R}$

  $f(0)$, $f(1)$, $f(10000)$ make sense

  $f(1/2)$ doesn't make sense

  $f(-5)$ technically shouldn't either

- Typically subscripts are used for inputs

  Instead of $f(0), f(1), f(2), \ldots$

  see $f_0, f_1, f_2, \ldots$

- Can be patterned

$$\underset{a_0}{\underline{2}}, \underset{a_1}{\underline{6}}, \underset{a_2}{\underline{18}}, \underline{54}, \underline{162}, \_\_, \_\_, \_\_$$

$a_n = 3 \cdot a_{n-1}$ geometric sequence, common ratio $= 3$

$$\underline{2}, \underline{7}, \underline{9}, \underline{16}, \underline{25}, \underline{41}, \_\_, \_\_, \_\_$$

$$a_n = a_{n-1} + a_{n-2}$$

$$\underline{3}, \underline{3}, \underline{3}, \underline{3}, \underline{3}, \_\_, \_\_, \_\_$$ constant seq.

$$\underline{2}, \underline{7}, \underline{12}, \underline{17}, \underline{22}, \_\_, \_\_, \_\_$$

$a_n = a_{n-1} + 5$ arithmetic sequence, common difference 5

Unpatterned

$$\underline{3}, \underline{1}, \underline{4}, \underline{1}, \underline{5}, \underline{9}, \_\_, \_\_, \_\_$$

## Arithmetic

- Have form $a_n = a_{n-1} + d$ (recursive formula)

$$= (a_{n-2} + d) + d = a_{n-2} + 2d$$

$$= a_{n-3} + 3d$$

$$= \cdots$$

$$= a_0 + nd. \quad \text{(closed formula)}$$

Ex.) $a_0 = 15$, $d = -7$

$$a_{31} = 15 + (31)(-7)$$

- Summing terms in an arithmetic sequence

Ex.)

$$\begin{array}{rcl} 11 + 18 + 25 + 32 + 39 + \cdots + 67 & = & S \\ 67 + 60 + 53 + 46 + 39 + \cdots + 11 & = & S \\ \hline 78 + 78 + 78 + \cdots + 78 & = & 2S \end{array}$$

$$\Rightarrow S = \left(\frac{78}{2}\right)(\text{\# of terms}) = \left(\frac{\text{first} + \text{last}}{2}\right)(\text{\# of terms})$$

Generally: $a_0 + a_1 + a_2 + \cdots + a_n$ ($a$'s from arithmetic sequence)

$$= \left(\frac{a_0 + a_n}{2}\right)(n+1)$$

## Geometric

- Pattern (recursive formula)

$$a_n = a_{n-1} \cdot r$$

$$= a_{n-2} \cdot r \cdot r = a_{n-2} \cdot r^2$$

$$= a_{n-3} \cdot r^3 = \cdots = a_0 r^n.$$

- Summing terms of a geometric series

$$a_0 + a_1 + a_2 + \cdots + \underline{a_{n-1}} = a_0 + a_0 \cdot r + a_0 r^2 + \cdots + a_0 r^{n-1}$$

$$S = a_0 + a_0 \cdot r + a_0 r^2 + \cdots + a_0 r^{n-1}$$ ← multiply both sides by r

$$rS = \quad a_0 r + a_0 \cdot r^2 + a_0 r^3 + \cdots + a_0 r^n$$

subtract

$$S - rS = a_0 - a_0 r^n$$ factor $$(1-r)S = a_0(1-r^n)$$

Divide $$S = a_0 \frac{1-r^n}{1-r}.$$

Ex. 1

$$2 + 6 + 18 + 54 + 162 + 486 + 1458 = 2 \cdot \frac{1-3^7}{1-3}$$

$a_0 = 2$

$r = 3$

$n = 7$