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Math 251, Fri 24-Sep-2021 -- Fri 24-Sep-2021
Discrete Mathematics
Fall 2021

Friday, September 24th 2021

Wk 4, Fr
Topic:: Countable and uncountable infinity
[redacted]
Read:: Rosen 2.5

Say you borrow 10K at 8% interest, compounded monthly.
  If at the end of each month you pay $250,
    how much will you owe after one month?

    how much will you owe after n months?

More on cardinality: (2.5)
 - one-to-one correspondences and cardinality
    When there exists f:A -> B, a bijection, then |A| = |B|.

 - Facts:
    1. If f:A -> B is one-to-one, then |A| \le |B|. $\le$

        arrow diagram, finite sets

        when A \subseteq B, |A| \le |B|
          finite and infinite set examples

2. Schröder-Bernstein Theorem

obvious for finite sets

infinite sets?
positive evens and Z+
naturals and the integers

- Infinity is infinity is infinity ... or is it?
  define "countable set"

  Cantor's proof there is an uncountable set

8% compounded monthly $\longrightarrow$ $\frac{1}{12}(0.08)$ is assessed each month.

$\Rightarrow$ at end of month

$$(\text{old balance})\overbrace{\left(1+\frac{0.08}{12}\right)}^{r} = \text{new balance}$$

outset owe $10000 = a_0$

End of 1^{st} month $(10000-250)(1.0067) = a_1 = (a_0-250)r$

End of 2^{nd} month: $a_2 = (a_1 - 250)r$

$$= \left[(a_0-250)r - 250\right]r$$

$$= a_0 r^2 - 250r^2 - 250r$$

End of 3^{rd} month:

$$a_3 = a_0 r^3 - 250\left(r^3 + r^2 + r\right)$$

$\vdots$

After n months:

$$a_n = a_0 r^n - 250\underbrace{\left(r^n + r^{n-1} + \cdots + r\right)}_{\text{terms in a geometric sequence.}}$$

$$\underbrace{r + r^2 + \cdots + r^n}_{n \text{ terms}} = r \cdot \frac{1-r^n}{1-r}$$

common ratio r

1^{st} term r

$$a_n = a_0 r^n - 250r \cdot \frac{1-r^n}{1-r}$$

closed formula

What about adding infinitely many terms of a geometric

$$a_0 + a_0 r + a_0 r^2 + a_0 r^3 + \cdots + a_0 r^n + \cdots = a_0 \cdot \frac{1}{1-r}$$

true iff $|r| < 1$.

Ex.)

$$3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \cdots$$

$$= 3 \cdot \underbrace{\left(1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \cdots\right)}_{a_0 = 1,\ r = 1/2}$$

parentheses $= 1 \cdot \frac{1}{1 - 1/2} = 1 \cdot \frac{1}{1/2} = 2$

Sum of all terms

$$3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \cdots = 3 \cdot 2 = 6.$$

Ex.)

$$0.32\overline{9} = 3 \cdot \frac{1}{10} + 2 \cdot \frac{1}{10^2} + 9 \cdot \frac{1}{10^3} + 9 \cdot \frac{1}{10^4} + \cdots$$

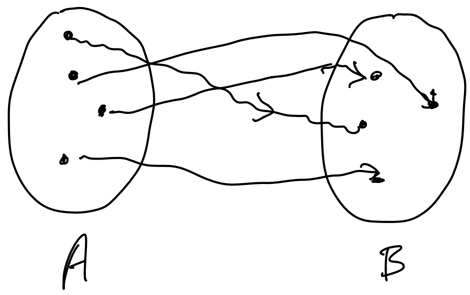
$$= 0.3 + 0.02 + \underbrace{\left[\frac{9}{10^3} \cdot 1 + \frac{9}{10^3} \cdot \frac{1}{10} + \frac{9}{10^3} \cdot \left(\frac{1}{10}\right)^2 + \cdots\right]}$$

$a_0 = \frac{9}{10^3}$, $r = 1/10$

Sums to

$$\frac{9}{10^3} \cdot \frac{1}{1 - 1/10} = \frac{9}{10^3} \cdot \frac{10}{9} = \frac{1}{10^2}$$

$$0.32\overline{9} = 0.3 + 0.02 + \frac{1}{10^2} = \underline{0.33}$$



Ex.] Naturals $\overset{A}{\mathbb{N}}$, $\overset{B}{\text{nonnegative evens}}$

$\{0, 1, 2, 3, \dots\}$ $\{0, 2, 4, 6, \dots\}$

$B \subseteq A \Rightarrow |B| \leq |A|$

Check out $f : A \to B$ given by $f(n) = 2n$

A		B
0	$\to$	0
1	$\to$	2
2	$\to$	4
3	$\to$	6
$\vdots$		$\vdots$

injection from A to B $\to$ $|A| \leq |B|$

Schröder – Bernstein

$|A| = |B|$.

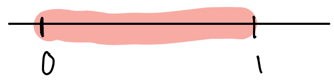
Defn: A set is called countable if

- its cardinality is finite, or
- it can be put in 1-1 correspondence w/ $\mathbb{N}$.

Examples:

1. $A = \{1, 2, 3\}$.
2. $\mathbb{N}$
3. $\{0, 2, 4, 6, \dots\}$
4. $\mathbb{Z}$
5. $\mathbb{Q}$

Nonexample: $(0,1)$.



There are different sizes of infinity

$$|\mathbb{N}| = |\mathbb{Q}| < |(0,1)| = |\mathbb{R}|$$