Math 251, Fri 1-Oct-2021 -- Fri 1-Oct-2021
Discrete Mathematics
Fall 2021

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Friday, October 1st 2021

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Wk 5, Fr
Topic:: Comparison of functions
Read:: Rosen 3.2

Bubble sort

Unordered list: 31, 22, 67, 42, 11, 38

1st pass: 31, 22, 67, 42, 11, 38

31 > 22 : swap

22, 31, 67, 42, 11, 38

22, 31, 67, 42, 11, 38

67 > 42 : swap

22, 31, 42, 67, 11, 38

67 > 11 : swap

67 > 38 : swap

22, 31, 42, 11, 38, 67

First pass over

still needs sorting

largest entry in place

list had 6 entries, took 5 comparisons for the pass.

Pass 2: unsorted portion of list has 5 entries, needs 4 comparisons

... and so on, for 5 passes through the list

Total # of comparisons

$5 + 4 + 3 + 2 + 1 = 15$

Q: What if list size were $10,000,000$

# of comparisons: $1 + 2 + 3 + \cdots + 9,999,999$

$$= \left(\begin{array}{c}\text{avg. of} \\ 1^{st} \text{ and last}\end{array}\right)\left(\#\text{ of terms}\right)$$

$$= \left(5000000\right)\left(9,999,999\right)$$

Estimate: each comparison requires $t$ microsecs.

$\rightarrow$ $(t)(\#\text{ of comparisons})$ estimates run time

list has $n$ entries

$$\text{total \# of comparisons} = 1 + 2 + 3 + \cdots + (n-1)$$

$$= \left(\frac{n}{2}\right)(n-1) = \frac{1}{2}n^2 - \frac{1}{2}n$$

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Sequences as recurrences:

$a_n = a_{n-1} + 7$ arithmetic

$a_n = 3a_{n-1}$ geometric

$a_n = 2a_{n-1} - a_{n-2}, \quad a_0 = 4, \; a_1 = -3$

$4, \; -3, \; 11$

$a_2 = 2(a_1) - a_0 = 2(4) - (-3) = 11$

In closed form

Use iterative approach to solve the recurrence
find closed form solution

Ex.] $a_n = a_{n-1} + 17, \quad a_0 = -5$

$= (a_{n-2} + 17) + 17 = a_{n-2} + 2(17)$

$= (a_{n-3} + 17) + 2(17) = a_{n-3} + 3(17)$

$= \ldots = a_0 + n(17)$

$= 17n - 5$ soln. (closed form)