

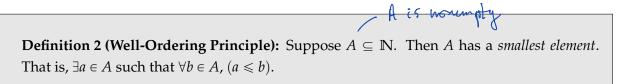
Math 251, Wed 20-Oct-2021 -- Wed 20-Oct-2021 Discrete Mathematics Fall 2021

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Wednesday, October 20th 2021 -----

Wk 8, We Topic:: Euclidean algorithm Read:: Rosen 5.2

We have two equivalent notions of induction. A third notion that looks different but is nonetheless equivalent is the **Well-Ordering principle**.



Note that the set {positive real numbers} does not have a smallest element, but that this does not violate the well-ordering principle.



3. A simple polygon with  $n \ge 3$  sides can be triangulated into n - 2 triangles (use strong mathematical induction, and the fact that every simple polygon with at least four sides has an interior diagonal).

$$\frac{MAIH 251 Notes}{Division Migorithm}$$
4. Given any integer n and any positive integer d, there exist integers (r) and (r) such that  $n = dq + r$   
and  $0 \le r < d$  (use the well-ordering principle).  
Consider the set  

$$A = \{n - dq : q \in \mathbb{Z} \} \cap \mathbb{N}$$

$$If n = 3(1, l = 17)$$

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$$S_0 + Le Will-Ordering 136$$

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$$If n = 3(1, l = 17)$$

$$If n = 3$$

MATH 251 Notes

5.3: Recursive definitions ! Structural Induction **Examples of recursive definitions** basis step 1. arithmetic and geometric sequences  $a_{0} = 51$ ,  $a_{n} = a_{n-1} + 23$  $g_{0} = -2$ ,  $a_{n} = (a_{n-1})3$  - recursive step 2. Fibonacci numbers f. = 0, f.= 1 (buis step)  $f_n = F_{n-1} + f_{n-2}$  (removine step)  $a_{0}$   $a_{1}$   $a_{2}$   $a_{3}$ 1 1 2 6 3.  $a_0 = 1$ , and  $a_n = n * a_{n-1}$  for  $n \in \mathbb{Z}^+$ 24 126 720 int **function** fact(int n) if n==0 return 1 **else return** n∗fact(n−1)  $W = \{T, F, p, q, \dots\}$ 4. Define the set  $\mathcal{W}$  as follows: **basis step**: T, F, and p, where p is a propositional variable, are in W. **recursive step**: If  $E, F \in \mathcal{W}$  then each of  $(\neg E), (E \land F), (E \lor F), (E \to F)$ , and  $(E \leftrightarrow F)$ are in  $\mathcal{W}$ . Call *W* the set of **well-formed formulae** in propositional logic. Why are  $\wedge pq$  and  $\neg \wedge p \vee q$ not well-formed?

5. Consider set *S* of strings formed this way:

aaaaaa bbbb

**basis step**: the letter *a* is a string in *S* **recursive step**: for each  $x \in S$ , *ax* and *xb* are in *S*.

6. Take  $\Sigma$  be some set of allowable characters. Define  $\lambda$  to be the empty string, the string containing no characters. We can think of the set  $\Sigma^*$  of strings over the alphabet  $\Sigma$  as defined inductively:

basis step: 
$$\lambda \in \Sigma^*$$
  $\Sigma^* = \{ a, f \}$   
recursive step: If  $w \in \Sigma^*$  and  $x \in \Sigma$ , then  $wx \in \Sigma^*$ .  
 $\Sigma^* : \{ a, b, \cdots, \mathcal{F} \}$  after one recursive step  
After many,  $\Sigma^*$  contains strings of any length of chars. From  $\Sigma$ 

7. Define the **length** function for inputs from  $\Sigma^*$  recursively:

**basis step:**  $\ell(\lambda) = 0$  (length of empty string) **recursive step:** For  $w \in \Sigma^*$  and  $x \in \Sigma$ ,  $\ell(wx) = \ell(w) + 1$ . (Here wx is the concatenation of w followed by x.)

8. Consider the Cantor middle-thirds set defined recursively as

**basis step**: Start with the full interval of real numbers  $C_0 = [0, 1] = \{x \mid 0 \le x \le 1\}$ . **recursive step**: For each unbroken subinterval *I* still in  $C_n$ , divide *I* into three parts of equal length:  $I = [a, b] \cup (b, c) \cup [c, d]$ , and include only  $[a, b] \cup [c, d]$  in  $C_{n+1}$ .

9. We define recursively various collections of **rooted trees**. Let the set  $\mathcal{R}$  be defined as follows:

**basis step**: A single vertex  $r \in \mathcal{R}$ .

**recursive step**: Suppose  $T_1, T_2, ..., T_n \in \mathcal{R}$  are disjoint having roots  $r_1, ..., r_n$ , respectively. The graph formed by taking as root a vertex r not in any of  $T_1, ..., T_n$ , and adding an edge from r to each of the vertices  $r_1, ..., r_n$  is also in  $\mathcal{R}$ . Call the resulting tree  $T_1 \cdot T_2 \cdots T_n$ .