Math 251, Mon 8-Nov-2021 -- Mon 8-Nov-2021

Discrete Mathematics

Fall 2021

Monday, November 8th 2021

Wk 11, Mo

Topic:: Recurrences

Read:: Rosen 8.1

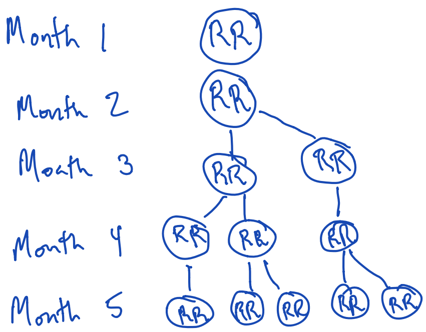
Counting problems:

- various things like _nP_r, _nC_r: from Ch.6/M252, not studied in this course
- modeling using Recurrence relations

Ch. 8, Section 2, 1st half has been covered so far, providing a method for solving linear, homogeneous, k^{th}-degree recurrence relations with constant coeffs.

Modeling problems: use recurrences

1. **Posed by Leonardo of Pisa**: A pair of rabbits does not breed until it is 2 months old. At age 2 months, they begin producing a pair of offspring every month. Use recurrence to track the number R_n of pairs of rabbits after n months.



R_n = number of pairs of rabbits after n months

$R_1 = 1$

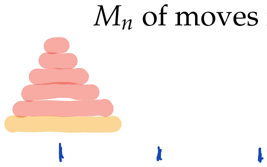
$R_2 = 1$

$R_3 = 2 = R_1 + R_2$

$R_4 = 3 = R_2 + R_3$

Sensible to write $R_n = R_{n-1} + R_{n-2}$?

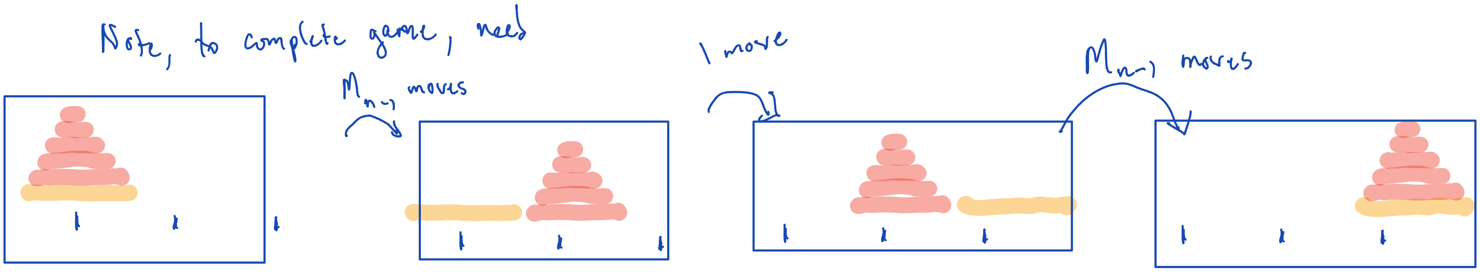
2. **Tower of Hanoi**: see http://www.mathsisfun.com/games/towerofhanoi.html
Must move a tapered stack of rings to a different pole, moving only one ring at a time, and never placing a larger ring over a smaller one. Use recurrence to track the minimum number M_n of moves in order to win game with n rings.



Let M_n = minimum # of moves needed to win beginning w/ n rings

$M_1 = 1$ Recursive formula: $M_n = M_{n-1} + 1 + M_{n-1} = 2M_{n-1} + 1$.

Note, to complete game, need



3. Let b_n represent the number of bit strings (strings of 0s and 1s) of length n not containing consecutive 0s. Write a recurrence for b_n.

n = 1: 0 1

n = 2: 01 10 11

$b_1 = 2$, $b_2 = 3$

$b_n = b_{n-2} + b_{n-1}$

When forming a valid bit string of length n
- take valid bit string of length (n-1) and tack on a '1' at the end.
- take a valid bit string on length (n-2) and tack on '10'.

4. **Enumerating codewords**: Say a valid codeword is a string from the alphabet "0–9" containing an even number of 0s. Let a_n represent the number of valid codewords of length n. Write a recurrence for a_n.

Valid: 00002 (5-digit word) Invalid: 01

12345 10200

$a_1 = 9$

$a_2 = 82$

Method fails to count some valid words such as 1010.

1. One approach: a valid word is obtainable taking a valid word of length n-2 and either
   - tacking on 00
   - tacking on any of 81 pairs of 11, 12, ..., 19, 21, ..., 29, ..., 91, ..., 99

Suggests $a_n = 82 a_{n-2}$

2. Every valid (n-1) length code word can have a 1–9 tacked onto it and every invalid (n-1) length code word can have a 0 tacked onto it.

a_{n-1} of these

$$a_n = 9a_{n-1} + (10^{n-1} - a_{n-1})$$