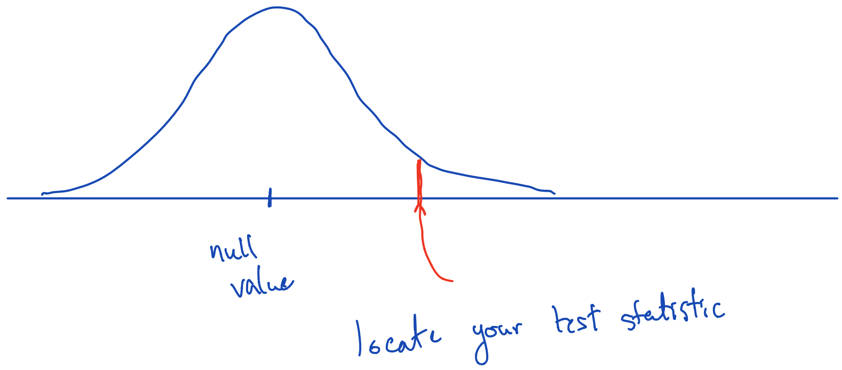
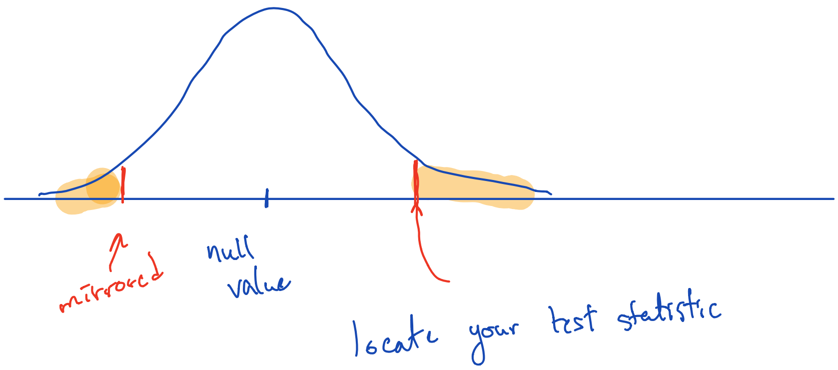
Usual situation for randomization distribution hypothesis test

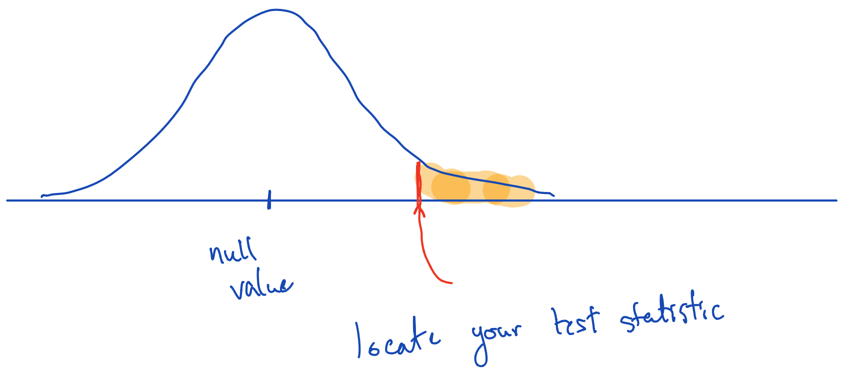


Determining your P-value



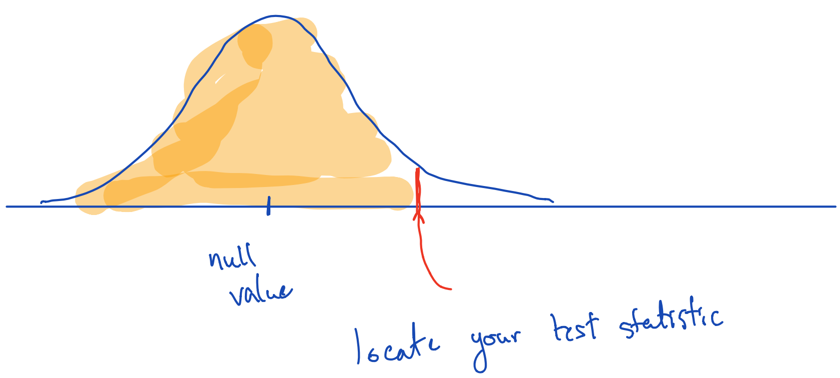
Case w/ 2-sided alternative hyp.

H_a: param. $\neq$ null value



1-sided, right-tailed alternative

H_a: param. $>$ null value



1-sided, left-tailed alternative

H_a: param. $<$ null value

$\Rightarrow$ P-value is larger than 0.5

4.63

Stat 145, Fri 8-Oct-2021 -- Fri 8-Oct-2021
Biostatistics
Spring 2021

Valeria, Jeffrey, Lauren, Josiah

Friday, October 8th 2021

Wk 6, Fr
Topic:: Randomization distributions
Topic:: Matched pairs data
Read:: Lock5 4.4
HW((PS08
HW((PS09 Ellenberg

} look for green entries in calendar to come

Some homework hints:
- How to understand the data given in Problem 4.54
- How to understand the data given in Problem 4.63
  the nature of one-sided alternative hypotheses and their P-values
- How to understand the data given in Problem 4.64
  asymmetric null distributions, and the policy of doubling one tail

Hypotheses tests involving bivariate data
- nature of data when looking at
  the difference of two independent (i.e., not matched pairs) group means

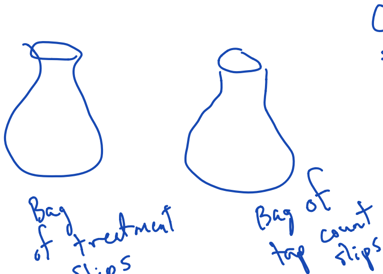
Treatment	Taps
Caffeine	246
Caffeine	248
Placebo	242
⋮	⋮

Caffeine: μ_c, $\bar{x}_c$ computed from sample of caffeine

Placebo: μ_p, $\bar{x}_p$ " " " " placebo

$H_0: \mu_c - \mu_p = 0$

$H_a: \mu_c - \mu_p > 0$



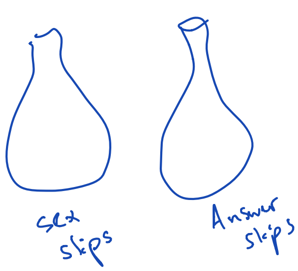
  the difference of two (independent) group proportions

Sex	Answer
F	No
F	Yes
M	Yes
M	Yes
M	No
⋮	⋮

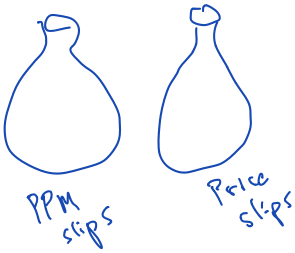
$H_0: p_M - p_F = 0$

$H_a: p_M - p_F \neq 0$

test stat $\hat{p}_M - \hat{p}_F$



slope/correlation for linear relationship of two quant vars



PPM(x)	Price(y)
2.7	79
3.4	299
2.8	199
⋮	⋮

H_0: either $\rho = 0$ or $\beta_1 = 0$

H_a: $\rho > 0$

- how randomization samples in cases mentioned above are carried out so (p.266)
  1. we are consistent with the null hypothesis

     Most important indicator of this: that its mean is the null value
  2. we use only data from the original sample
  3. we reflect the way the data were collected.

Mental models for what is being done:

two bags, slips drawn without replacement, building new pairings

two decks of cards, one shuffled

Note: These bivariate hypothesis tests are firsts of sampling w/o replacement

See farther down for more on the distinction between "matched pairs" and "2 independent samples"

Exception: matched pairs

- null and alternative hypotheses $H_0: \mu_{Diff} = 0$ vs. $H_a: \mu_{Diff} \neq 0$ (2-sided version) talk about "$\underbrace{\text{mean difference}}_{\mu_{Diff}}$" as opposed to "$\underbrace{\text{difference of means}}_{\mu_1 - \mu_2}$"

- do like univariate quantitative case

    first compute "column" of differences (only relevant variable henceforward)

    take a bootstrap dist., then translate it so centered at 0

Two independent samples vs. matched pairs

Consider this research question: Is it better to fish a certain lake from shore, or from a boat?

Our response variable will be quantitative, the ratio of fishing hours to fish caught. Here is some data.

month	Apr.	May	June	July	Aug.	Sept.	Oct.
shore	3.3	3.6	3.9	3.2	3.0	1.8	1.6
boat	3.8	3.0	3.3	2.2	1.6	1.4	1.5

We have a binary categorical explanatory variable: "Where fishing from?", with values "shore" and "boat". We have a quantitative response variable. We have bootstrapped and tested hypotheses for the difference $\mu_1 - \mu_2$, but the methods I've discussed have presumed *independent samples*. The data collected to investigate the question do not represent independent samples. The responses in the different months are naturally related: when one goes up, the other seems more likely to go up, both being related to the population of fish in the lake during that month. This data is **matched pairs** data, and we should:

- use the months as *cases*, and produce for each case a single difference:

month	Apr.	May	June	July	Aug.	Sept.	Oct.
shore	3.3	3.6	3.9	3.2	3.0	1.8	1.6
boat	3.8	3.0	3.3	2.2	1.6	1.4	1.5
difference	-0.5	0.6	0.6	1.0	1.4	0.4	0.1

- Proceed as if in a "single mean" setting. A confidence interval would be for the purpose of estimating the mean difference μ_{diff}. An hypothesis test would focus on hypotheses:

$$\mathbf{H}_0 : \mu_{\text{diff}} = 0 \qquad \text{vs.} \qquad \mathbf{H}_0 : \mu_{\text{diff}} \neq 0.$$

  Either way, the slips of paper we would insert into a bag for bootstrapping or randomization would contain only the last set of numbers, the differences.

Practice: Does the data suggest independent samples, warranting analysis on the difference of two means $\mu_1 - \mu_2$, or is it matched pairs?

1. A study was conducted to investigate the effectiveness of hypnotism in reducing pain. Results for randomly selected subjects are shown in the table below. A lower score indicates less pain.

subject	A	B	C	D	E	F	G	H
before	6.6	6.5	9.0	10.3	11.3	8.1	6.3	11.6
after	6.8	2.4	7.4	8.5	8.1	6.1	3.4	2.0

2. To study the effects of a drug on blood pressure, patients had a base reading taken of their diastolic blood pressure. After 3 weeks on the medication, new readings of their diastolic blood pressures were taken.

3. A collection of statistics students is randomly assigned to two groups. One group is given a study regimen that includes listening to recordings of classical music by Mozart, while the other group must study in silence. The response variable is student scores on an exam.