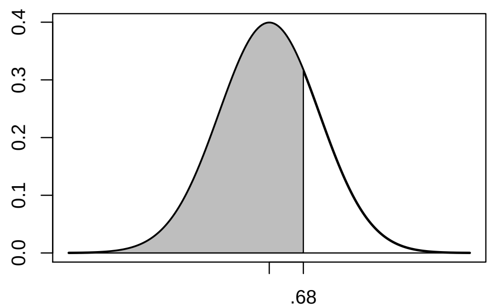
MATH 143: Introduction to Probability and Statistics

Worksheet for Fri., Oct. 14: Normal Distribution

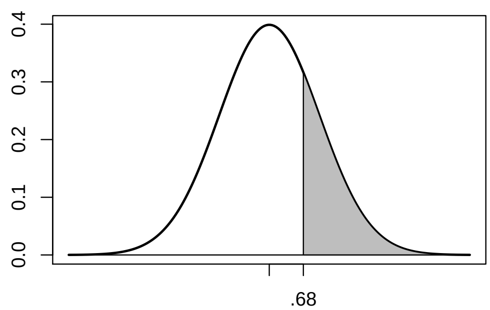
Practice using the standard normal table to find the following. In each case sketch the area that you are looking for under the standard normal curve drawn. (It is always a good idea to draw such a sketch, partly to remind yourself that probabilities correspond to areas, partly because the sketch can help you to figure out how to use the table correctly, and partly because it allows you to check visually whether your answer seems reasonable.)

1. The proportion of Z-values less than 0.68—that is, $\Pr[Z < 0.68]$. How about $\Pr[Z \leq 0.68]$?



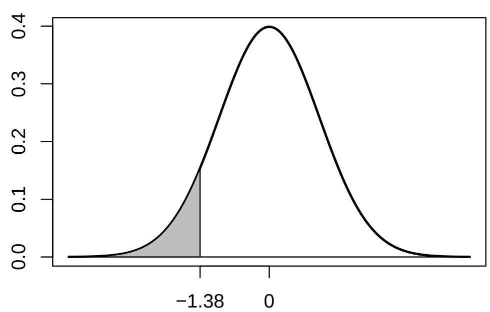

```
> pnorm(0.68, 0, 1)
[1] 0.7517478
```

2. $\Pr[Z > 0.68]$



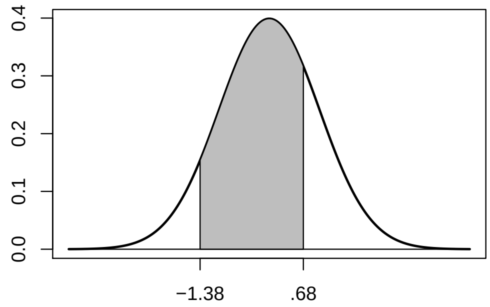

```
> 1 - pnorm(0.68, 0, 1)
[1] 0.2482522
```

3. $\Pr[Z < -1.38]$



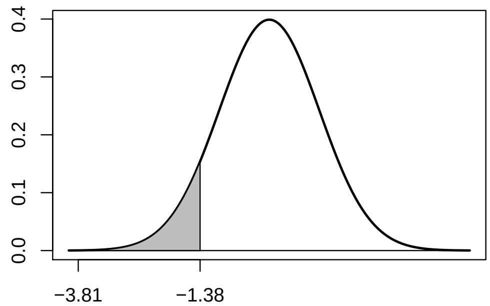

```
> pnorm(-1.38, 0, 1)
[1] 0.08379332
```

4. $\Pr[-1.38 < Z < 0.68]$



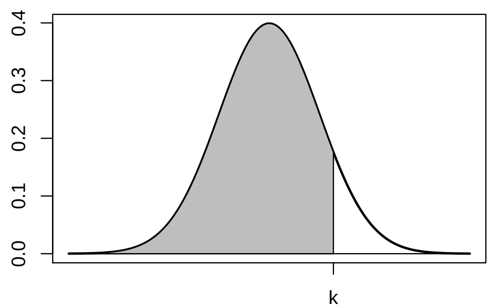

```
> pnorm(0.68) - pnorm(-1.38)
[1] 0.6679544
```

5. $\Pr[-3.81 < Z < -1.38]$




```
> pnorm(-1.38) - pnorm(-3.81)
[1] 0.08372384
```

6. Find the value of k such that $\Pr[Z < k] = 0.8997$.




```
> qnorm(0.8997, 0, 1)
[1] 1.279844
```

7. Find `Q1` and `Q3` (the 1st and 3rd quartiles) for the standard normal distribution.

```
> qnorm(0.25)
[1] -0.6744898
> qnorm(0.75)
[1] 0.6744898
```

The Empirical Rule. Let us continue to use Z to denote a variable with a standard normal distribution. Use the table of standard normal probabilities to find:

8. $\Pr[-1 < Z < 1]$

```
> pnorm(1) - pnorm(-1)

[1] 0.6826895
```

9. $\Pr[-2 < Z < 2]$

```
> pnorm(2) - pnorm(-2)

[1] 0.9544997
```

10. $\Pr[-3 < Z < 3]$

```
> pnorm(3) - pnorm(-3)

[1] 0.9973002
```

Critical Values. Now use the table of standard normal probabilities "in reverse" to find as accurately as possible the values of z^* satisfying:

11. $\Pr[Z > z^*] = 0.10$

```
> qnorm(0.9)

[1] 1.281552
```

12. $\Pr[Z > z^*] = 0.05$

```
> qnorm(0.95)

[1] 1.644854
```

13. $\Pr[Z > z^*] = 0.01$

```
> qnorm(0.99)

[1] 2.326348
```

14. $\Pr[Z > z^*] = 0.001$

```
> qnorm(0.999)

[1] 3.090232
```

Package Weights. Suppose that the wrapper of a candy bar lists its weight as 8 ounces. The actual weights of individual candy bars naturally vary to some extent, however. Suppose that these actual weights vary according to a normal distribution with mean $\mu = 8.3$ ounces and standard deviation $\sigma = 0.125$ ounces.

15. What proportion of the candy bars weigh less than the advertized 8 ounces?

```
> pnorm(8, 8.3, 0.125)

[1] 0.008197536
```

16. What proportion of the candy bars weigh more than 8.5 ounces?

```
> 1 - pnorm(8.5, 8.3, 0.125)
[1] 0.05479929
> pnorm(8.5, 8.3, 0.125, lower.tail = F)
[1] 0.05479929
```

17. What is the weight such that only 1 candy bar in 1000 weighs less than that amount?

```
> qnorm(0.001, 8.3, 0.125)
[1] 7.913721
```

18. If the manufacturer wants to adjust the production process so that only 1 candy bar in 1000 weights less than the advertised weight, what should the mean of the actual weights be (assuming that the standard deviation of the weights remains 0.125 ounces)?

```
> 8 - qnorm(0.001, 0, 1) * 0.125
[1] 8.386279
```

19. If the manufacturer wants to adjust the production process so that the mean remains at 8.3 ounces but only 1 candy bar in 1000 weighs less than the advertised weight, how small does the standard deviation of the weights need to be?

```
> (8 - 8.3)/qnorm(0.001, 0, 1)
[1] 0.09708008
```